THE IMPACT OF USING COMPUTER ALGEBRA SYSTEMS (CAS) IN TEACHING AND LEARNING OF “DOUBLE INTEGRAL”

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Abstract
The topic of double integral is one of the most important and key concepts of multi-variable calculus. In this study, the effectiveness of Computer Algebra Systems (MAPLE12) in the development of procedural and conceptual knowledge in teaching-learning double integral for students was investigated. By using a simple random sampling technique forty-four students among Shahid Rajaei Teacher Training University students were chosen as experimental and control groups. The experimental group was taught using CAS software while the control group was taught through traditional methods. Both groups were given a pre-test on the topic of integral in order to find out about students’ previously acquired knowledge. The results showed no statistically significant differences on the students’ pre-test scores between the two groups.

After teaching the topic, the post-test was given to both groups to measure acquired procedural and conceptual knowledge about the topic. At the end, interviews with randomly selected students from experimental group were done in order to find out about students’ thinking process. Achievement levels have been determined to specify students’ understanding of procedural and conceptual knowledge. Results clearly indicated that using CAS software enhanced the students’ achievement.

Introduction
Integral has been taught to students in high school and in the first year of university. According to some instructors some entrants into mathematics degrees and engineering students cannot grasp the concept of it very well, mainly for those integrals which involve finding volumes and areas. Above all, students cannot visualize the outcome of a solid created by revolving a region about the axes. Sealey (2006) and Kouropatov (2008) illustrate that some students do not grasp the concept of the integral and mostly they use formulae to solve the integral problems.

Holding an interview with a few mathematics educators and academic staff at the Shahid Rajaei Teacher Training University, the researcher found out that the visualization of three dimensional shapes is one of the major difficulties for the students. Undoubtedly, it is impossible to accurately plot these shapes on the whiteboard. Considering the existing problems in teaching the double integral concept, this study attempts to explore the effectiveness of CAS in learning and teaching of this concept.

Theoretical Framework
Mathematics education is one of the branches of science and epistemology. New teaching approaches indicate the importance of thinking and reasoning, understanding concepts, solving problem and the ways of processing them, as well as emphasizing the learners’ engagement in the process of learning-teaching. Some affecting factors in teaching-learning include sociology, psychology and technology.

Based on constructivist learning theory, if a learner constructs a concept such as mathematics concept through an active role at the same time as experimenting, conjecturing, proving and applying, they make the knowledge by themselves instead of receiving it. In conjunction with changing the methods of teaching and learning, this approach emphasizes the construction of the knowledge by the learner. Kim (2005) states:

There are three fundamental differences between constructivist teaching and other teaching approaches. Firstly, learning is an active constructive process rather than the process of knowledge acquisition. Secondly, teaching is supporting the learner’s constructive processing of understanding rather than delivering the information to the learner. Thirdly, teaching is a learning-teaching concept rather than a teaching-learning concept (p. 9).

National Council of Teachers of Mathematics (2000) indicates the importance of using multiple representations in learning and teaching mathematical concepts and relations and also introduces it as one of the standards of mathematics education. In addition to multiple representations, visualization also plays an important role in comprehending the concepts of multivariable calculus. Therefore, the role of visualization is
very significant in learning and teaching of multivariable calculus. Bishop (1989) states “An interactive computer environment, particularly when dynamic visual image are employed, can encourage and to some extent develop the pupils’ visualization abilities” (p.13).

Active method is one of the teaching methods which correspond with constructivist approach. The Computer Algebra Systems’ (CAS) capability of visualization and conversion of symbolic, graphical, and algebraic representation helps the learners to build their knowledge in an active and constructive manner.

Sociology is known as another affecting factor in mathematics education. In this regard, one of the theories is Vygotsky’s theory. Generally, Vygotsky’s theory consists of three major concepts: (1) the importance of culture and social aspects; (2) emphasis on the significant role of language: and (3) the concept of Zone of Proximal Development (ZPD). In the Vygotsky point of view individuals acquire the application of the sign or “psychological tool” from face to face communication and observation of other individuals. Development in learning occurs when these application become internalized, providing learning scaffolding which is more tangible in the beginning of learning rather than during or at the end of the learning. Therefore, the initial support provides the opportunity in which the learners are able to continue their learning independently through building on their knowledge. Analyzing the scaffolding process of Zone of Proximal Development, Vygotsky argues that there is a distance or limitation between what an individual performs without others’ help and what he does with their guidance. This limitation is called the domain of development performance (Meadow, 1996). Vygotsky’s theory in relation to ZPD may be fulfilled with technology. Kim (2005) writes “Computer programs can be designed by the expert to help the students reach their potential in their ZPD in many ways (p. 9).”

Vygotsky’s theory described that learning requires semiotic mediation which internalizes the external concepts.

…the sign (or “psychological tool”) acts as an instrument of psychological activity in a manner analogous to the role of technical tool in the laboratory. The divergence is that the function of the technical tool is externally oriented: it must lead to changes in objects. The sign, on the other hand, is internally oriented: it’s intended to master the person himself. The process of genesis of sign, called by Vygotski “process of internalisation”, is characterized by the “internal reconstruction of an external operation” and by the “transformation of an interpersonal process into an intrapersonal one” (cited in Falcado, 2002, p. 2).

Based on the Vygotskanian theory of the role of semiotic mediation in learning and teaching, instruments are one part of the process of constructing meaning. CAS as an instrument in learning and teaching mathematics has the capability of performing various acts. It provides us with various solutions to the problems and development of understanding of the basic concepts of calculus. Heid (1988) states “Computing devices are natural tools for the refocusing of the mathematics curriculum on concepts.” (p. 4). It is worth noting that the instrument and tool on its own does not have any impact, however, by providing a rich-learning environment for its use, it can be applied as a facilitator of learning and teaching processes. One way to enhancing this environment is to design the activities suited to learners. As a matter of fact, it can be concluded that CAS plays the role of an external instrument which is internalized by the learner during the interactive process. Consequently, the instrument aids learners in development of their visualization.

The aim of designing these activities is to teach either concepts or procedures or both of them. Conceptual knowledge is seen as the knowledge of the core concepts and principles and their interrelations in a certain domain and procedural knowledge is seen as the knowledge of operators and the conditions under which these can be used to reach certain goals (Byrnes & Wasik, 1991).

In this regard, Haapasalo and Kodegevich (2000) discuss two pedagogical approaches: developmental and educational. Developmental approach indicates the development of conceptual knowledge by the procedural knowledge which relates to simultaneous activation view, while the educational approach point out the development of procedural knowledge by conceptual knowledge which is supported by dynamic interaction view. Byrnes and Wasik (1991) stated that simultaneous activation view means that conceptual knowledge is necessary for procedural knowledge but is not enough, i.e. conceptual knowledge is developing before procedural knowledge. Therefore, it is possible to design the activities so that they make simultaneous activation between concept and procedure.

**Literature on Use of CAS in the Process of Teaching-Learning**

From 50 years ago, when computers were used for the first time in education, CAS was first introduced (Awi et al., 2007, p. 221). Computer Algebra Systems entered into education from 1970 and it was used in the classrooms in 1980. muMath was the first CAS which was executed on personal computers, which was replaced by the other software such as Matlab (1979), Maple (1980), Mathematica (1980) and Mathcad (1990). Arnold (1996) explains such a trend has arisen for the portable calculators too. In the early 1980 the simple calculators were developed, in the later 1980’s more complicated graphic calculators, and in 1995 the calculators were equipped by CAS.
Computer Algebra Systems can carry out all the routine mathematical processes such as accurate computations, plotting the graphs, computations by vectors and matrix, statistical calculation, simplification of algebraic expressions, as well as computation of derivations and routine symbolic integrals. Furthermore, it has some extra facilities such as plotting three dimensional animated graphs and presenting multiple forms of representation. Thomas, Monaghan and Pierce (2004) discussed about different kinds of CAS support with emphasis upon concepts, generalization, and mathematical modeling. One of the other applications of CAS that has been illustrated by Ball and Stacey (2005) is that makes the problem solving procedures quicker and gives more instruction time to the learners and, as a result, they are more willing to symbolize the mathematical functions. Aydm (2005) shows graphic representation and simulation by CAS increases the presentation of the teachers and it provides the students immediate feedback and also gives them some examples to clarify how they can solve the equations. Consequently, CAS makes the mathematics more interesting and enjoyable for them.

Meagher (2005) explains that from the time when CAS was introduced, research on its impact in mathematics education has been developed in two main strands as follow:

The first has concentrated on the effectiveness of technology in supporting the learning of specific topics (e.g. solving equations, differentiation, integration, application in optimization) ... The second strand examines technology-enhanced curriculum design suggesting new topics (e.g. cryptography, chaos theory) (p. 3).

Meagher (2005) investigates the effectiveness of CAS (Mathematica) on the students’ learning in calculus. His study focuses on the question “What are the processes of learning in a Computer Algebra System (CAS) environment for students learning calculus?” He applied the Rotman Model of Mathematical Reasoning as a macro-framework and the Pirie-Kieren Model of Mathematical Understanding as a micro-framework. Rotman Model used for understanding the place of technology in the learning of mathematics. The Pirie-Kieren Model used as a lens through which to interpret and analyze specific learning episodes as they take place in the classroom. The two frameworks together provided a vehicle for understanding learners’ mathematical activity, mathematical reasoning and mathematical development in a CAS environment across a period of time. He indicated that introduction of technology at the beginning of the instruction period impacts significantly on student behavior and while students experience in a CAS environment, probably there is a need for intervention of educators to develop the experimental behavior and strategies.

Tokpah (2008) has done a meta-analysis to investigate the effectiveness of CAS, in comparison to non-CAS instruction, on students’ achievement in mathematics at post-secondary and pre-college instruction. He states that a great deal of research has been conducted on the effectiveness of the technology, in general, and CAS in particular. Tokpah describes the research by Hollard and Norwood (2005) who compared the usefulness of CAS with traditional instruction and other researchers such as Brown (2007) did not apparently show what type of education for the control group was used (seems students in the control group did not use CAS). Tokpa found regardless of how CAS were used students contributed to a significant increase in their performance.

The Review of the Literature on the Maple

Maple is one of the comprehensive and powerful computer algebra systems, and was designed by the Symbolic Computation Group (SCG) at Waterloo University of Canada in 1981. Maple is extensively used in differentiation, integration, simple and partial differentiation equations, linear algebra, multivariable and vector computations, complex numbers, statistics, combinatorics, theory of numbers.

Aksoy and Bulut (2004) examine the effect of Maple software on the development of conceptual and procedural knowledge of the derivative concept for first-year students. They found out that students who were taught using Maple had improvement in conceptual understanding in comparison with those who received traditional instruction. However, these students did not show any differences in procedural understanding. Awi et al. (2007) also study the effectiveness of Maple on the learning and teaching of the application of the integral. They concluded that there is a significant difference between those students who work with Maple and those instructed by traditional methods. They stated that use of technology had enhanced the students’ achievement.

Noinang et al. (2008) used Maple as an instrument for learning and teaching multiple integrals. They showed that using Maple in mathematics education increases students’ interest in mathematics and helped them to improve their skills in self-assessment, self-planned learning, and self-motivation. Various facilities of Maple for mathematics education, including solving simple and double integral step by step, plotting the functions in two and three dimensions and, animations, encouraged researcher to apply Maple in instruction of the multiple integrals.

The researcher selected Maple for this study based on the following reasons. First, it involves an appropriate interactive environment for learning. Second, it has the capability of changing the parameters and showing animated Riemann sums for the simple and double integrals. Third, it has the capability of multiple forms representation and visualization of three dimensional curves, surfaces, and their revolving and intersections, as well as rotation of plots in order to investigate the curves from different directions and
representing them on the 2-dimensional xy-coordinate plane. All above mentioned characteristics of Maple assist students in reaching faster and more effective learning. On the other hand, this software involves a manual which not only provides students with sufficient examples but also makes Maple user friendly.

**Methodology**

The participants in this study consisted of 44 first-year students at the Shahid Rajaee Teacher Training University. The present study was conducted to investigate the effectiveness of CAS (MAPLE 12) on teaching and learning of the double integral concept.

It is a quantitative study based on an experimental design as illustrated in Table 1.

### Table 1

<table>
<thead>
<tr>
<th>Group</th>
<th>Pre-test</th>
<th>Independent Variable</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>experimental</td>
<td>T1</td>
<td>X</td>
<td>T2</td>
</tr>
<tr>
<td>control</td>
<td>T1</td>
<td>--</td>
<td>T2</td>
</tr>
</tbody>
</table>

In this design, T1 and T2 are pre-test and post-test and X is the independent variable which in this study is the use of Maple.

Most first-year students at this university take a three-credit calculus (II) at the second semester. It was presented for nine classes and two of them were chosen randomly. Then, the students were divided in two groups of 22 people. Students in the experimental group and the control group used the same textbook with different methods. While the control group received the instruction using traditional approaches without using Maple, the experimental group was taught using Maple 12. However, active method of teaching was deployed for both groups. Both groups attended one and half hours per week in six sessions for learning and teaching. All participants were given a pre-test, measuring their acquired knowledge from the previous knowledge. At the end a post-test was administered for both groups.

All of the activities were designed based on a constructivist approach (see Appendix 1). The initial activities concentrated on making hypotheses, conjecturing, investigating and testing them in order to present the proposed patterns by students. Also these activities aimed at developing visualization and generalization abilities of the students.

The double integral concept was taught to both groups in two following steps: Firstly, simple integral concept is reviewed and then the simple integral concept in the 2-dimensional coordinate plane was generalized to the double integral concept in three-dimensional space. The designed activities were used for the purpose of procedural and conceptual learning of the integral concept.

Daugherty and Funke (1998) and Owston (1997) indicated that “It is known that when students experience difficulty in accessing technology, such as installing software on their computers, they are discouraged from using computers in their learning” (cited in Aminifar et al., 2006, p. 2). Therefore, the researcher provided some interactive Maple worksheets (see Appendix 2) to guide students during the process of learning and teaching, in order to face the possibility of students having problems with Maple12. These interactive worksheets prepared using Maple, animated and non-animated graphics plotted by Maple and Maplets in teaching integral. Since students can change the parameters and observe the changes simultaneously, they understand the integral concept more effectively. Therefore, students have the opportunity of various experiments which give them better understanding of the subject.

**Data Analysis**

By using descriptive and inferential methods the data was analyzed to test the hypotheses. Analysis of the data obtained from interviews and class observations verified the hypotheses. In particular, the researcher proposes these hypotheses:

- Experimental group performs better than control group in conceptual knowledge.
- Experimental group performs better than control group in procedural knowledge.

Box’s test of equality of covariance matrices shows covariance matrices are equal (F = 1.940, p > 0.05) and Levene’s test of equality of error variances shows that variances for the two dependent variables are equal ($F_{\text{conceptual}} = 1.780, p > 0.05$ & $F_{\text{procedural}} = 1.244, p > 0.05$).
Table 2
Descriptive Statistics

<table>
<thead>
<tr>
<th>Group</th>
<th>Mean</th>
<th>Std</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>post-test: conceptual</td>
<td>0.00</td>
<td>1.00</td>
<td>22</td>
</tr>
<tr>
<td>post-test: procedural</td>
<td>0.00</td>
<td>1.00</td>
<td>22</td>
</tr>
<tr>
<td>Total</td>
<td>11.27</td>
<td>11.14</td>
<td>22</td>
</tr>
<tr>
<td>Total</td>
<td>7.14</td>
<td>7.45</td>
<td>22</td>
</tr>
<tr>
<td>Total</td>
<td>7.30</td>
<td>2.32</td>
<td>44</td>
</tr>
</tbody>
</table>

Table 3
Tests of Between-Subjects Effects

<table>
<thead>
<tr>
<th>Source</th>
<th>depended variable</th>
<th>Type III sum of squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>Difference of conceptual scores</td>
<td>46.023</td>
<td>1</td>
<td>46.023</td>
<td>5.432</td>
<td>0.025</td>
</tr>
<tr>
<td>Group</td>
<td>Difference of procedural scores</td>
<td>42.023</td>
<td>1</td>
<td>42.023</td>
<td>6.278</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Analysis of Quantitative Data

After gathering data MONOVA test was used to investigate the difference between control and experimental groups in conceptual and procedural understanding.

In this paper, the conceptual and procedural scores were considered as the dependant variables and teaching by CAS was regarded as the independent variable. Table 2 presents the descriptive statistics and results from Multivariable tests for Pillai’s Trace show the independent variable impacts on the linear combination of two dependent variables ($F = 4.543, p < 0.05$). In Table 3 in order to check the effects of two dependent variables, differences between conceptual scores and also differences between procedural scores (both in pre-test and post-test) was calculated. In both of them the p-value is less than 0.05, which means that independent variable effects on dependent variables (i.e. conceptual and procedural scores) are significant.

The results reveal that the experimental group performs better in conceptual and procedural understanding than the control group. This finding corresponds with other research conducted on the effectiveness of CAS in learning-teaching integral such as Awi et al. (2007) and Noiing et al. (2008). Specifically, the results of this study are similar with the findings of Bulut and Aksoy (2004) who investigated the effectiveness of CAS in conceptual and procedural knowledge.

Analysis of Qualitative Data

The class observation was performed during learning and teaching. In this regard, observations of students’ activities were recorded when they practiced with the simple and double integral concepts. Then to investigate the students’ perceptions in depth and also to examine their viewpoints about Maple12 software, ten students were selected randomly to be interviewed by the researcher. Most students found using Maple12 was useful in learning multivariable calculus. They emphasized the significant role of opportunities to visualize during instruction of the double integral and stated that while they had a lot of difficulties in visualization of surfaces and three dimensional shapes before the instruction, using this software assisted them in looking at surfaces and their intersections with other shapes in various directions. Therefore, this provided them with a better understanding of volume between two surfaces and establishing the integration of the limits. Students provided the following explanations for this preference:

- Maple helped me to facilitate visualization in three dimension forms.
- Using the Maple software is so helpful to solve the complicated integral. When it was required to do the integration, sometimes it was better to swap the order of integral to solve it by Maple. Thus, once [the] Maple shows the complicated process of solving a problem by applying one of these ways, I completely perceive the necessity of changing the order.
- Maple solves the simple and double integral step by step which helps me to have a deep understanding of integrals.

At the time of using CAS, students easily computed the various integrations by themselves in the interactive environment of Maple in order to learn simple and double integral. Then they changed the parameters, and concurrently compared the different Riemann sums. Initially students investigated their conjectures and, after that, they stated regarding the hypotheses served. Activities were designed so that students would attempt the problems without CAS and then by CAS. Sarah described the procedure of solving the double integral problems as following:

When I encountered the double integral concepts, my mind performs like maple. That is, firstly, I plotted the shape in my mind, then I rotated it in different directions, next I solve it algebraically...
the help of maple, I get profound understanding of the different types of integral. Maple aids me in the spatial perception of the three-dimensional shapes.

Additionally, students asserted that compared to their previous traditional courses they had, they not only had more concentration on the concepts, but also they often participated in class discussions. Consequently, their self-confidence increased. Overall, students developed more positive self-motivation towards mathematics.

These findings of the study have generally agreed with some results of the CAS-CAT project that Flynn (2005) describes.

Discussion and Conclusion

In the past three decades, the development of CAS has led to advancements in the learning mathematics. On the one hand, CAS enhances students’ understanding by converting algebraic representation into graphic and symbolic ones at the same time. On the other hand, it plays an important role in visualization and multiple forms of representation as a facilitator, particularly in teaching the basic concepts of multivariable calculus. Furthermore, giving immediate feedback to the students supports their stating and testing hypotheses and also improves the students in presenting and investigating the patterns in conjunction with the adjustment and generalization of these patterns.

The sequence of activities with the help of the interactive Maple environment in learning and teaching of double integral concept entails the step by step analysis of mathematical concepts, in conjunction with accurate visualization of three dimensional shapes and multiple representations of double integral problems.

Students are able to notice that changing one representation resulted in changes in the other. Consequently, by understanding the connections among these representations, they get more profound understanding of different types of integral.

Moreover, it is interesting and pleasing for the students to use Maple with animated graphics. In using Maple a cycle is offered for the conceptual and procedural understanding. It means that conceptualization and subsequent representation of computational methods and solving various problems led the students to obtain superior understanding of the double integral. This conceptual understanding develops procedural understanding and solving problems brings about more inclusive conceptualization of the double integral.

It seems that if the tool is properly utilized in the teaching and learning mathematics, it provides students with more self-confidence and higher positive attitude towards mathematics.

The findings of the present study reveals that using CAS is effective and valuable not only in learning and teaching the double integral concept but also in development of basic concepts in multivariable calculus. It increases students’ achievement and they make greater benefits from conceptual understanding of double integral concept. Therefore, this helps them improve in procedural understanding and achieve a better result. It is hoped that active methods would be used to integrate CAS in teaching and learning and provided students with proper activities.

References


**Appendix 1**

Given \( f = -2x^2 + 4x + 6, x \in [0,3] \). Use Maple and estimate the area with Riemann sums by having six points (n = 6) of: left sum; right sum; midpoints; random points; upper sums and lower sums.

What does it impact choosing left and right sum, midpoints and random points, upper and lower sum in your answer? Conjecture and then illustrate it.

Which of the above options is near to actual integral? What is the reason for your hypotheses?

Now increase number of points, for example n = 8 then n = 10, etc. What does it change? Write results of each part and then compare them with each other. Conjecture and test your hypotheses. Explain your worked solution geometrically. Can you present a pattern? What do you find by changing and increasing points?
Consider another function, for example \( f = \sin x \), then repeat above steps and compare results. While \( n \) increases present a pattern for Riemann sums based on your understanding of the above exercises.

**Appendix 2**

Below is an example of interactive Maple worksheet.