CASE STUDY: WHAT DOES THE EQUAL SIGN MEAN TO CHILDREN?

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Abstract:

What an equal sign mean to a child might look trivial but it is a pertinent concept. The meaning of an equal sign is useful for the conceptual understanding of algebraic equations. In this paper, the main focus is to illustrate on a case study on the interpretation of the equal sign by a child in the elementary level. The clinical interview method contains several tasks to probe the understanding of the child on the equal sign. It is important for a child to know the different meanings of the equal sign that was used. In this case study, the child had a strong sense of the equal sign as an operator. However, in situations involving words and money, the child within the age group of elementary school level regarded the equal sign as a relational symbol between quantities.

Keywords: Equal sign, relational understanding, children

It is very interesting to read many articles on the meaning of the equal sign as interpreted by children. This captured my attention and made me interested to find out more about it. The curiosity of understanding what an equal sign means to a child seems trivial but in reality it is a pertinent concept. I have decided to use the clinical interview method to explore what a child knew about the equal sign. By having a one-to-one interaction with a child, I can ask and clarify the meaning of the equal sign with him or her. Children who seem to be good in mathematics do not necessarily have relational understanding of the equal sign. I have selected the meaning of the equal sign to find out children's understanding because it is useful to explore their conceptual understanding of the algebraic equations. If children do not have a strong relational understanding in the equal sign at elementary level, they might face difficulty in solving algebraic equations in middle school (Knuth, McNeil, & Alibali, 2006).

The main purpose of the interview was to find out what does the equal sign mean to a child. In addition, I also explored the interpretation of the equal sign by the child. The lack of understanding of the equal sign symbol among children, were mentioned in many studies (Baroody & Ginsburg, 1983, Behr et al., 1980; Kieran, 1981; Rittle-Johnson & Alibali, 1999; Seo & Ginsburg, 2003). As we know the understanding of the equal sign in the elementary level is very critical for the child. Most elementary school children regard the equal sign as an operation and not a symbol of mathematical equivalence (Seo & Ginsburg, 2003). Generally the equal sign is divided into two categories, firstly as an operational symbol and secondly as a relational symbol. Many studies done had concluded that children typically regard the equal sign as the "answer to", "it equals to" and "the end is coming up"(Baroody & Ginsburg, 1983; Behr, et al., 1980; Seo & Ginsburg, 2003).

The understanding of the equal sign did help children solve algebraic equations in middle school (Knuth, McNeil, & Alibali, 2006). We can say that the poor understanding of the equal sign in the middle school is due to the lack of attention given in its elementary curricular. Kieran (1992) mentioned that "one of the requirements for generating and

adequately interpreting structural representations such as equations is a conception of the 'left-right equivalence' of the equal sign" (p. 398).

Background information

My clinical interview subject was a third grader Hispanic student, anonymously given the pseudonym as Justin. He was 9 years old and was currently a student of the elementary school PS 125 in Manhattan, New York. He was supposed to be in grade four this year but due to his poor result, he was detained in Grade 3. He seemed to like mathematics very much and enjoyed solving challenging problems. After speaking to his parents, they informed that he was an average student. However he had deep interest in mathematics. Justin was active in sports and loved solving puzzles. He could also communicate well with me. He seemed playful but when a Rubik's cube was given to him, he was able to solve it.

Before the study, I had sought the permission of Justin's parents to conduct this interview with him. They were obliged with my request and had no objections for me to do this clinical interview. They signed the parent consent form and asked me for the duration of the interview. I told them the interview session would take about 30 minutes. In addition, I also explained to them the purpose of the interview and how it could help Justin.

Clinical Interview Protocol

The tasks with interviewing the child involved 3 sections:

a) First section involved asking the child the meaning of the equal sign.

The child was shown the equal symbol and the following questions were raised:

i) What is the name of the symbol?

ii) What do you understand with the symbol above?

iii) Explain the meaning of the symbol.

b) Secondly, some examples on 'addition and subtraction' were shown. Then the child was also enquired about the meaning of the equal sign

For example: '3 + 2 = 5'

'4 – 1 = 3'

i) What do you understand with meaning of the equal sign in these two operations?

ii) What do you understand with meaning of the equal sign in the example above?

Then I showed '5 = 3 +2' followed with questions:

iii) What is the meaning of the equal sign here?

iv) Is the addition above correct?

c) Thirdly the interpretation of the child on equal sign that involved money was sought after.

The following equation was shown '1 dollar = 100 pennies', followed with the questions:

i) What does the equal sign mean here?

ii) Can the symbol mean anything else? If yes, please explain.

Next another example was shown: '1 dollar = 4 quarters', followed with the questions:

a) What does the equal sign mean here?

b) Can the symbol mean anything else? If yes, please explain

Findings and interpretations

One of the objectives of this interview was to determine what the equal sign meant to Justin. I also wanted to know the interpretation of the equal sign and how he understood it.

In the beginning of the interview, I started out by writing the equal symbol. I asked Justin, "What is the name of this symbol?" Justin responded by saying that it was the equal sign. He further explained that it was used when 5+5 equals a number which is 10. When asked further, Justin wrote down two words, "house=house" and said that it equalled each other. Then he wrote "5 + 10 = 15". He explained to me that the equal sign showed the number on both side was and were equalled to. I enquired further and asked "What do mean by number on the side?" He could not answer but responded with "It shows the number and what does it equal to." I presumed he meant the number 5 and 10 as he pointed towards it. Then Justin said "Equal sign could be used in different ways in words and numbers."

To test his operational understanding of the equal sign, I then showed him the equation 15 = 5 + 10 and asked "What do you understand with the equal sign here?" as I pointed towards the equal sign. Justin responded by saying "It equals the same number but it is backwards." He further explained that counting backwards was like counting from 15, 14, 13 till 5. Then I asked him on the equal sign in "5+ 10 = 15" and "15=5+10", "Any difference?" Justin said "No, but it's like the same in the two equations." Justin seemed to understand that the equal sign could be used in two different contexts in the case of 15=5 +10 where his answer could be interpreted explicitly as 15 parts broken into 5 parts and 10 parts and 5+10 = 15 where 5 parts and 10 parts gave a total of 15 parts. Perhaps Justin was able to see the equivalence relation with these two examples.

Next I showed Justin "7 – 3 = 4", and asked him the role of the equal sign. Justin said, "It is the same but with different numbers." I continued with showing him "5 + 10 = 15" and "7 – 3 = 4" and then pointing at the equal sign in the two equations with question on the meaning of the equal sign. He responded by saying "It has a same meaning. Equals in take away is counting down while equals in addition is counting up." Justin showed two different interpretations for the equal sign here. I presumed he got confused with the meaning of the equal sign compared to the addition and subtraction sign.

Moving on, I wanted to know Justin's relational understanding of the equal sign. I showed him "1 dollar = 100 pennies" and asked "What does the equal sign mean here?". He answered with a smile, "It means how much it is ~~it~~ but with words. It is the same but with dollars and pennies." I then asked him to write for me an example how an equal sign could be used. He wrote "2 X 3 = 6." Justin then said "The equal sign here means equal the number six. Equal sign can be used in many different ways in mathematics." When enquired further about his

understanding by many different ways, Justin said, “Three ways to use equal sign are in words, numbers and money.” I then said “Can the equal sign be used for other things?”. I asked him to write it down. Justin wrote “100 pennies = 1 dollar” and said “It means the same thing.” Then he said “A shirt is 100 dollars and another shirt is 100 dollars. Later he wrote “100+ 100 = 200.” He then showed me another example “20 – 2 = 18 and ten + ten = 20.”

After showing me a few examples, I asked Justin the same question “What does the equal sign mean in your examples?” Justin said, “Count it and the equal sign makes the number equal or ….” When asked, “Can it be the same?” Justin responded “No. Two words that are spelt the same use the equal sign.” To further test his relational understanding, I showed him “Blue = Green”. He quickly responded and said “No, they are not the same.” He then wrote “Blue= Blue.” Justin demonstrated a strong equivalence understanding of the equal sign when dealing with words.

Overall, Justin had an operational understanding for the equal sign. He seemed to be able to explain the meaning of the equal sign as an operator in the addition and subtraction equation. Justin’s interpretation of the equal sign seemed to indicate the answer is or it is equalled to. This showed that he only knew the operator meaning of the equal sign. He was able to understand the use of the equal sign in the transitive way of 15 = 5 + 10 and the usual way of 5+10=15.

However, he still did not get the relational meaning of the equal sign clearly. In contexts related to money, he seemed to have some relational understanding. What surprised me was that Justin was able to give the usage of the equal sign in 3 categories. Perhaps he interpreted the symbol differently for words, operations and money. Obviously Justin had two different interpretations. The first idea was the equal sign acts as an operator symbol for addition and subtraction. Secondly the equal sign was used as a relational meaning in other contexts such as word, colour and money situations.

After reviewing this clinical interview, I noticed that I could have asked more probing questions to Justin. One example was when he said “No, it is like the same.” I could have pressed him further and asked, “Can you explain what is ‘the same’ means?” I should also have been more patient for Justin to deliver his answers in certain parts of the interview before asking another question. One other area I could improve was to reduce the tendency to give leading questions as I usually wanted the student to get a correct answer. Perhaps the lack of experience in adapting and adjusting to the interview caused it. But this will further help me improve my interviewing techniques in the future.

Conclusion

It is important for a child to know the different meanings of the equal sign. In this case study, the child had a strong sense of the equal sign as an operator. Nevertheless, in situations involving words and money, the child regarded the equal sign as a relational symbol between quantities. In the many studies done, a children will face difficulty in learning the algebraic concepts if they do not have a strong relational understanding during their elementary level. More emphasis should be given by teachers on the different meanings of the equal sign. It can be done by using more concrete objects and relating it to the everyday life. This will help the child relate the different interpretations of the equal sign and make connections between them. Kids like Justin would probably benefit if this is done consistently with the stress on the relational and conceptual understanding of the equal sign.

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