CREATIVE MATHEMATICS EDUCATION SUPPORTED BY DIGITAL TOOLS THAT ENHANCE STEM LITERACY AND STEM RELATED CAREERS

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Abstract

In this article, the author presents topics and methods for computer-aided mathematics activity or high school mathematics research. There is no standard method to cultivate mathematical creativity, but the author thinks that his method and the material in this article can be a standard way to train students how to be creative in mathematics. His method starts with some problems chosen for the students. The author teaches students a number of mathematical problems as open-ended topics. Students are expected to change these problems using the method suggested in this article, and present some new problems. Then students start to study these problems using computer algebra system Mathematica that is available online for free. It is often the case that students discover some new things in the output of the mathematics software. The author used this method many times during the last 20 years. The author's students were able to discover new formulas and theorems through appropriate use of computers, digital tools and e-platforms. The author and students had published the results of the research in more than thirty refereed papers, and talked about their research in 11 countries. Moreover, some students were able to further their investigation to become computer programmers, engineers and other STEM related careers. In conclusion, the topics and methods used are appropriate and effective. The topics presented here remain effective for further high school mathematics research, and the author hopes that teachers will attempt to use in their classroom.

Keywords: Mathematics activity; Creativity; High school mathematics research; Computer algebra system; Mathematica

Introduction

In this article, the author writes about his experience in promoting creative mathematics education supported by digital tools that enhance STEM literacy of his students, many of whom were subsequently involved in STEM related careers.

Background and Rationale

The author introduces himself, and talks about his activity on mathematics education as a secondary mathematics teacher. He taught an elementary probability theory in a class for 12th grade students, and gave students some problems. At the end of lesson, he asked them to change the problems by themselves. On the next day, one of his students came to him with a very elegant formula of mathematics. He thought that his student's formula is far simpler and more elegant than the formula he presented in his Ph.D. thesis. Then he began to do mathematics research with his high school students. Through his research with students he got awards and recognition both at national and international levels.

In many countries, teachers and researchers in education are looking for good ways to cultivate creativity. Although the need for courseware to cultivate creativity is very high, many people consider it difficult to develop. However, the author considers that it is not difficult to cultivate creativity, and his approach is very simple. In his view, the best way to cultivate creativity is to give students the opportunity to engage in very original activities, i.e., to create something. As students begin to create things regularly, it is natural to conclude that their creativity has been cultivated.

Aims and Focus of Study

In this article, the author presents a simple method for a high school mathematics research project. The author and their students have discovered new theorems and mathematical formulas using this method. The author has also taught students from other schools, and these students were also able to discover interesting mathematical facts. These results show the effectiveness of their method. In the workshop as reported by Miyadera (2017), the author taught teachers how to do mathematics research, so a large part of the exemplars presented in this article was based on the output of activities implemented in the workshop.

The author also considers mathematics to be a very good subject for students to create new things. In the world of science, an idea is good only when it is consistent with the laws of nature, and students are not free to propose new ideas. On the other hand, in mathematics, students are free to present ideas if those ideas are mathematically correct. After studying the proposed problem, we can evaluate the problem. If the answer of the problem is interesting, the problem is a good one. If the problem can be generalized to be a very difficult problem, then the problem is mathematically proper topic to study.

Methodology

How to Conduct Mathematical Research: A Method to Discover New Formulas

Here, the author presents a method of high school mathematics research in seven steps.

i. The teachers select a number of problems that seem interesting to students. In this regard, there are a number of well-known problem books and puzzle books, for example, Dorichenko (2012). Usually, these books are used to learn how to solve problems, but they are also good for selecting problems for high school research. Mathematical games are also good topics for high school students, since games are usually interesting for them (Berlekamp, Conway, & Guy, 1982).
ii. Students are asked to select one problem that they like, and to change the conditions of the problem. Some students may change the conditions slightly, while other students may change the conditions drastically.
iii. The teacher and students now have a number of problems that are variants of the original problem.
iv. Students and teachers select a problem to study. They study it with a pen and paper. Once they understand the mathematical structure of the problem, they create a computer program using the computer algebra system. In the case of the author, he used Mathematica that is freely available on the web as elaborated below, but the web version of Mathematica is slower than the desktop version.

Remark. Note that Mathematica can be used for free at Wolfram Programming Lab (https://lab.open.wolframcloud.com/app/). This is an important fact, since Mathematica is an expensive software to buy.

v. Through the calculations using the Mathematica program, students obtain data about the problem, and begin to study that data in an attempt to identify patterns. Usually, students are good at finding patterns.

vi. If students discover interesting patterns in the data, they begin to prove the existence of the patterns.

vii. If students manage to prove the existence of the patterns, they have new formulas and theorems. They can then write a paper and submit it to a contest for high school students. They can also submit it to a high school research journal or undergraduate journal. If the results are really impressive, they can even submit it to an academic journal for mathematicians.

Remark. If our aim is to cultivate creativity and you are not interested in mathematics research itself, you can stop when you discover a pattern in Step (v). It is often very difficult for high school students to prove a formula by themselves, and they need someone to help them. Sometimes they need a mathematician. If your aim is to do highschool mathematics research, then Step (vi) and Step (vii) are important.

Detailed Explanation of the Method

(a) One of the easiest ways to create a variant is to generalize the original problem. For example, changing the numbers in a problem can be a good way to generalize the problem. In the original problem of Example 1 to be elaborated, there are six cards, and two of which are red cards. One student discovered a very interesting formula by changing these numbers.

(b) Substituting a geometrical figure with another figure can also be a good method. When we create new problems, elementary probability theory and combinatorics are very good topics for research. A problem can be deemed good for high school research when computers can be used to study it. Using computers is easier for students than using precise mathematical reasoning. The history of computer-aided mathematics research is not long, even for professional mathematicians, and hence many mathematical facts remain to be discovered using computers.

(c) The following site can be used for help in mathematics. They are very helpful, for example, Ask Dr. Math available on URL http://www.washington.edu/doit/ask-dr-math-0

Analysis and Discussionon Exemplary Cases of Creative Mathematics Education

Example 1 (Pascal-Like Triangles)

The author taught elementary probability theory to 12th grade students.

The original problem. There is a box that contains six identically-sized cards, two of which are red and the others white. Two players A and B take turns to pick a card. The first player to draw out a red card loses the game, and the game ends. We calculated the probability that the first man will lose the game. The probability of A losing the game in the first round is $\frac{2}{6}$. We calculated the probability that A will lose the game in the third round. We note that A and B must have survived the first and second rounds respectively, and then A loses in the third round. This gives $\frac{4}{6}\times\frac{3}{5}\times\frac{2}{4}$. Similarly, the probability of A losing the game in the fifth round is $\frac{4}{6}\times\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}\times\frac{2}{2}$. From this, we see that the probability of A losing the game is $\frac{2}{6}+\frac{4}{6}\times\frac{3}{5}\times\frac{2}{4}+\frac{4}{6}\times\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}\times\frac{2}{2}=\frac{3}{5}$.

The first variant of the original problem. After teaching this problem to the students, the author suggested that it might be mathematically interesting to change the number of cards and the number of red cards. In fact, the author did not know if the problem would become more interesting with a different number of cards. One student studied this problem under the condition that there are identically-sized cards, and all of these cards are white except for *m* of them, which are red. He calculated the first player's probability of losing the game for *n* = 1, 2, 3, 4, 5 and *m* = 1, 2, …,*n*. The following Table 1 shows the probabilities of A's losing the game for *n* = 1, 2, 3, 4, 5 and *m* = 1, 2, … ,*n*.

Table 1

The Probabilities of A's Losing the Game for Natural Numbers n and m

	m = 1	m = 2	m = 3	m = 4	m = 5
n = 1	1 / 1				
n = 2	1 / 2	1			
n = 3	2 / 3	2 / 3	1		
n = 4	1 / 2	2 / 3	3 / 4	1	
n = 5	3 / 5	3 / 5	7 / 10	4 / 5	1

The student then arranged the fractions in Table 1 as a triangular shape, to create Figure 1. The structure of this triangle of fractions is, at first glance, not clear, but we know that it has a very elegant mathematical structure once we arrange it as in Figure 2. Regarding the pattern in the triangles, see the fractions enclosed by boxes in Figure 3. This reminds us of Pascal's triangle.

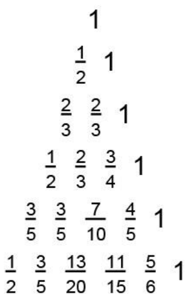

Figure 1. Fractions.

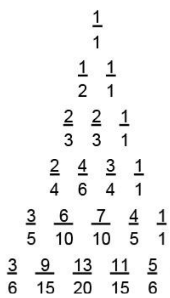

Figure2. Without Reduction.

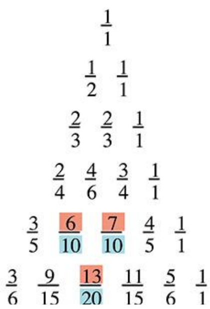

Figure 3. Properties.

The other variants. One student proposed a new idea. He thought that this game could be played with more than two players. Hence, we discovered that Pascal-like property still holds for this generalization. The original game ends as soon as a player draws a red card. We can extend the game to a second stage by continuing with the remaining *p*-1 players and *n*-1 cards in the box (*m*-1 of which are red). As soon as a player loses in the second stage, the remaining *p*-2 players can begin the third stage, and so on. The author and his students published the solution of this generalization in Matsui, Minematsu, Yamauchi, and Miyadera (2010). They are now studying another generalization of this problem.

Example 2 (The security of a combination of bicycle lock)
In Japan, many high school students travel to school by bicycle, and use some type of bicycle lock. The locks in Figures 4 and 5are typical bicycle locks. For the lock in Figure 4, you rotate the numbers to open it, and for the lock in Figure 5, you push four buttons to open it.

Figure 4. A type of bicycle lock.

Figure 5. Another type of bicycle lock.

The original problem. Suppose that you forget the code of your bicycle lock. You have to try all the combinations of numbers to open the lock. Which is more difficult to open – the lock in Figure 4 or that in Figure 5? The number of lock combinations in Figure 4is 10^4 = 10,000, and the number of lock combinations in Figure 5 is ${}_{10}C_4 = 210$. The students thus realized that the lock in Figure 4 is a great deal safer than that in Figure 5. The author then asked the students, "What can we do to make the lock in Figure 5 safer?"

The first variant. What can we do to make the lock in Figure 5 safer? One student told the author, "It would be better to make two types of lock. In one type of lock, you are supposed to push four buttons and in the other type of lock, five buttons. That way, you would not know how many buttons you were supposed to push." This is quite a good idea. Another student proposed making a lock without any numbers on it. Without numbers, it would be difficult to try all the combinations. Although this is a very good idea, the author discovered that there is already a lock of this type in the market. See the lock shown in Figure 6.

Figure 6. A bicycle lock without number.

Conclusion

Some advice for teachers who are willing to start high school mathematics research is to read this article and select a topic that seems interesting. Then, please begin mathematics activity and contact the author. Please do not try to accomplish something big. Novices can take advice from the author and in that respect, they do not have to travel a lonely path. Incomplete research results are acceptable for educational purposes. The author hopes that many teachers will join in cultivating creativity in mathematics activity. The author and his students are still very active, as evidenced in the author's newest result reported by Miyadera, Inoue, and Fukui(2016).

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