A Framework for Designing Comics-based Mathematics Instructional Materials

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Received first draft 21 April 2022. Received reports from first and second reviewers (24 June 2022 and 6 July 2022). Received revised draft 9 August 2022. **Accepted** to publish 19 December 2022.

Abstract

Purpose and Research Question - In this paper we propose a PATH framework for designing comics-based instructional material for classroom lesson enactment through conducting a literature review in mathematics education.

Methodology – Systematic review was made with a focus on the potential benefits of comics for education, in particular, on developing students' motivation for learning and facilitating their knowledge retention.

Findings – We further demonstrate with an exemplar the use of the framework in designing one set of comics-based instructional material for lower secondary mathematics lessons on mensuration.

Significance and Contribution in Line with Philosophy of LSM Journal – An exemplar of a comics-based instructional material designed according to the PATH-CoHANa framework.

Keywords: *Comics; Education; Mathematics; Design principles for comics instruction*

Introduction

Although Singapore students have performed well in mathematics in various international comparative studies such as in the Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA), their attitude towards the subject can be further improved. For example, among the student participants in TIMSS 2019 and 2015, 35% and 33% respectively of the eighth grade Singapore students reported that they disliked mathematics. In an earlier study by Fan et al. (2005), in their random sample of 1215 secondary school students, 37% did not enjoy attending mathematics lessons, with more than 50% of them disliked learning the subject. Specifically, for low-attaining students, classroom anecdotes about these students lacking interest in the subject which they found boring, difficult, and irrelevant to daily life are plentiful (e.g., Toh & Lui, 2014; Toh & Kaur, 2019). Further, Toh and Lui (2014) also found that in addressing the affective needs of the low-attaining students, many mathematics teachers had already started to use alternative teaching approaches such as cartoons and comics to motivate their students to learn mathematics, although these efforts were sporadic. There appeared to be few design principles based on pedagogical theories of using comics for instruction. This paper aims to present a framework for designing comic-based instructional material and provides an exemplar of how a set of comic-based materials is designed using the framework.

Although some conservative educators might see comics as a distraction to lessons, others have come to realize that comics could be potentially powerful educational tools that can develop students' interest in the subject (Cleaver, 2008). Since the 1940s, there have been studies investigating the use of comics as a pedagogical tool (e.g., Hutchinson, 1949; Sones, 1944). Recently in many national contexts, researchers have tried to encourage the integration of comics into the school's curriculum (e.g., Adnan et al., 2019; Affeldt et al., 2018; Gokbulut & Kus, 2019; Toh et al., 2019). The benefits of using comics

as a teaching tool have been shown in numerous empirical studies. For examples, comics have been shown to improve students' interest in the subjects (Morris et al., 2007; Musa et al., 2020), and can motivate students to read and learn Mathematics (Toh et al., 2018). Comics could also help to reduce the anxiety students have when learning a concept that they deem difficult, especially in mathematics (Gokbulut & Kus, 2019; Krishnan & Othman, 2016; Clair, 2018). Further, it has also been shown to be a powerful way to communicate ideas using an approach that is aesthetically pleasing and mathematically accurate (Saracco, 2020), and can aid in information retention, especially that of complex ideas (Wylie & Neeley, 2016).

Literature Review

Student Responses and Feedback

Numerous studies have documented encouragingly positive responses from students due to comics intervention, both cognitively and affectively. In this section, we reviewed some of these studies which gave us the ideas to synthesize the framework of the design principles.

Toh et al. (2019) designed a set of seven comics to replace traditional teaching materials on the topic of Percentages. They took inspiration from the hit novel *Shopaholics* and used shopping trips as a storyline for all the comics. The sets of comics aimed to engage students in mathematical tasks while stressing the relevance of mathematics in real-life. A trial was then carried out where the lesson package with the seven comics was used with three classes from three Singapore mainstream schools. They found that students reacted positively to the comics instructional package. They enjoyed the humour in the stories, which in turn enabled them to appreciate the relevance of mathematics in the real world, and encouraged them to want to read more about mathematics. In particular, the mathematics jokes in the comics' instructional material attracted their attention not only to the jokes but also to the mathematics content embedded in the comics.

In another study in which mathematics was integrated into the storyline of comics instructional package, Mamolo (2019) saw positive feedback from 425 students who took part in the study. It was reported that the students found the comics-embedded material easy to read and its content easy to learn. The same problem presented in comics has a lighter cognitive load for the students compared to it being presented using the usual prose with lengthy sentences.

Comics have also been used in the instruction of other subjects, which we took reference from. For example, in the lower secondary Chemistry laboratory lessons reported in Affeldt et al. (2018), the students were given science practical worksheets with embedded comics that contained real-world context relatable to the students. The students found the comics interesting and fun. As a result, they were more interested in the experiment. In addition, students liked the story and felt that they could relate to the characters of the comics. The students could also better comprehend the experimental instructions when presented in the comic style. However, some of them could not understand the instructions as it was oversimplified. There was only information, but no tasks for the students. On the other hand, some students appreciated that they needed to figure out the experiment for themselves and that not everything is explained to them. According to the study by Krishnan and Othman (2016), students subject to comics intervention had a faster recall of the scientific facts and gained a significant improvement in their higher-order thinking skills.

Not all studies on the use of comics for classroom instruction report positively on the use of comics for instruction. The lack of student improvement after comics intervention could be due to the nature of comics. Students do not need to think as much when they read comics, compared to reading textbooks (Muliyardi et al., 2005, as cited in Turyanto et al., 2019). Designing appropriate comics that stimulate students' curiosity and thinking is important for its usage in the classroom (Toh, 2009). Thus, it is worthwhile to look into the factors behind successful comics-based instructional materials.

Principles of design of Comics-based instructional material

Researchers have proposed how comics-based instructional materials can be designed. For example, Chu and Toh (2020) proposed the Theme, Storyline, Characters, Test (TSCT) framework to design comics for teaching in the primary-level mathematics classroom. The outstanding features of the TSCT framework include having an interesting storyline and relatable characters, as well as infusing humor into comics. While the framework focused on comics for younger students in primary schools, we believe that the ideas underpinning the framework are still largely relevant for students at the secondary level. The framework also stressed the importance of the comics being contextually and culturally relevant, in addition to the importance of the comics including relatable characters to the students. This essential feature is also echoed by Azamain et al. (2020).

Format. On the design of comics-based instructional materials, Naylor and Keogh (2013), based on their research on the use of science concept cartoons for classroom use, suggested some key features of a concept cartoon include being based on daily situations, containing blank speech bubbles to invite students to think about alternative ideas, including common misconceptions in the speech bubbles. Not only that, the designers could consider using colloquial language in the design of the comics, a point that is also echoed by Han and Toh (2019).

Earlier research conducted by Kabapinar (2009) also suggested that comics should be presented in the form of worksheets for them to be impactful on students' learning. When students are presented with comics in the form of worksheets, they could control their pace of reading and better comprehend the context of the concept cartoon. Positive benefits such as diminished classroom management problems were also found in the comics lessons.

Storyline. The use of an authentic storyline is one of the most prominent features that has resulted in positive learning outcomes among students in the comics intervention is the use of an authentic storyline. Comics can be used to present concepts in authentic daily situations that are relatable to students. The storyline should be designed in a way to trigger students' thinking. As an illustration, one common strategy to frame the story is to have different fictitious characters presenting different and even contrasting opinions about an issue. These opinions contain both correct conceptions and common misconceptions among students. This could trigger students' thinking and debate. This could perhaps address the concern raised by Turyanto et al. (2019) on the lack of student reasoning in comics intervention.

Method

In this paper, existing education literature related to comics intervention for classroom instruction was reviewed. A total of 47 sources, including journal articles and book chapters, were reviewed. The sources were found through a search on Google Scholar, as well as the bibliographical databases ResearchGate and Education Resources Information Center (ERIC). Block building and citation searching techniques were used, using keywords such as "education", "comics", "teaching", and "mathematics". Boolean operators were also used to narrow down the scope of the papers. Based on the title of the article, papers on empirical studies were analyzed. Next, we propose a framework for designing comics for mathematics instruction.

Core features of the comics

As revealed in the education literature, many studies have shown the potential of comics as an instructional tool. We also weighed against the studies which did not have a positive impact of comics on student learning. In analyzing the reports and summarizing their strategies, we propose to condense

our findings into the PATH (Problem-Solving, Authenticity, Text, Humour) framework for designing mathematics instructional material using comics.

Problem-solving. The designer of comics instructional package needs to be constantly reminded of the essence of mathematics education, that is, problem-solving. Mathematical problem-solving has been the core of the mathematics curriculum of many countries, including Singapore (e.g., Toh et al., 2008a; 2008b; 2013). The final goal of any mathematics instruction should aim to develop mathematical problem-solving skills among students. To be able to achieve the goal, the students should be presented with sufficiently challenging mathematical tasks (Kroll & Miller, 1993). Ultimately, one of the main goals of a problem-solving curriculum is in improving students' willingness and ability to tackle non-routine problems, sustain their perseverance when tackling challenging problems, and raise their awareness of the existence of multiple methods to solve a problem (Aydoğdu & Ayaz, 2008). From the teacher's perspective, assessing students' problem-solving ability allows a teacher to determine if they have fully understood and can apply mathematical concepts (Pardimin & Widodo, 2017).

Research has shown that comics can improve students' problem-solving abilities (e.g., Gokbulut & Kus, 2019; Suryatin & Sugiman, 2019; Widodo et al., 2018). In comparison to using words and diagrams, explanations of mathematical concepts using visuals such as cartoons can facilitate students' understanding of difficult concepts (Azamain et al., 2020; Bobek & Tversky, 2016). As Polya (1957) suggests in his four phases of problem-solving, understanding the problem is the first step. We believe that comics can be useful in simplifying complex different concepts, thereby making higher-order thinking accessible to students.

We also noted that many of the comics intervention reported in the research papers were part of research projects and were not sustained over a long period of time. The inclusion of questions in their comics, being part of some research projects, was to fulfill the objective of observing students' improved problem-solving. In our view, for comics to truly replace a traditional lesson with a textbook, problem-solving must be integrated into the comics package as part of the learning process. The inclusion of problem-solving also ensures that students are focusing on mathematics, rather than merely being drawn to comics, a problem faced by Turyanto et al. (2019). Problem-solving can be incorporated into the comics by providing questions after the comics. Similar to what Toh et al. (2019) did, every comic can be coupled with a set of practice questions with varying levels of scaffolding, with the scaffolding gradually diminishing in the later questions. In other words, these questions should have a gradual transition, from (A) problems that are set within the context of the comics, to (B) questions within a similar (but not identical) context, and eventually to (C) examination-type questions, or questions of a higher level of difficulty and even context-free. It should be noted that the questions of types (A) and (B) should sustain the interest of the students through the authenticity of the context, the next point in this discussion.

During the lesson enactment, the use of comics provides the affordance for more student interactions in a discussion about the comics and the mathematics embedded in the comics. Engaging students in group discussions are one way that teachers can provide a learning environment conducive to problem-solving (Santos-trigo, 1996). Student discussion resulting in differing opinions can cause them to defend or alter their ideas, thereby developing their critical thinking ability.

Authenticity. Fan et al. (2005) found in their study that most students perceived mathematics as an important subject, but ironically, only 64% of the students responded that they would apply Mathematics in their daily life after their graduation. Another similar finding on tertiary students was reported by Lim-Teo et al. (1999). We believe that the use of *authentic* learning could bridge the gap in the relevance of mathematics for the students.

Authentic learning is an instructional approach that focuses on complex real-world problems and their solutions (Lombardi, 2007). Under authentic learning, students are given authentic contexts, contexts that students can relate to in their daily lives. These authentic contexts reflect real-life usage of the knowledge they need to know (Herrington & Oliver, 2000). Authentic learning has been shown to improve students' academic performance (Newmann et al., 1996), and motivate and engage students (Jones et al., 2010; Willems & Gonzalez-DeHass, 2012). For example, Wery and Thomson (2013) found that students who can connect lesson tasks to the real world are more likely to be committed to solving the problem posed. Lastly, authentic learning has been shown to help students develop expert thinking, which is the ability to identify and tackle problems without a standard solution (Lombardi, 2007).

While it is impossible to recreate a truly authentic environment in class, the use of comics could approximate such a learning environment. The illustration and texts used in comics can help students visualize an authentic context that could be related to the daily life of the students. In addition, to make the comics as authentic as possible, the comics should have a storyline that students are familiar with, and can relate to. The storyline could be simple, such as having to pay goods and services tax (GST) after having a meal at a restaurant. Another way to make comics authentic is to have characters that are based on students. For example, Fahruddin et al. (2022) used stories involving a teacher and two students to allow students to relate to the story as if it was their own experience.

For classroom enactment of the lessons, the teacher could further enhance the authenticity of the learning process, e.g., teachers can also engage their students to role-play the fictitious characters who appear in the comics, and further invite them to modify the discourse within the same mathematical concept (e.g., Toh et al., 2016).

Text. Compared to lengthy sentences and paragraphs usually found in textbooks, students are likely to be more interested in reading comics. As Keogh and Naylor (1999) suggested, the texts within the comics should be as succinct, preferably in the form of dialogues between the fictitious characters. This ensures that even students with relatively low literacy skills will be able to understand the story and the embedded concepts. However, we are still cautioned by Affeldt et al. (2018) that even when writing in easy sentence structures and keeping words to the minimum, students may still be unmotivated to thoroughly read through the texts. Thus, the authenticity of the contexts of the comics, as described in the preceding section, is an important consideration in designing comics instructional package. Nevertheless, we believe it is still important that mathematics comics be designed to have relatively few texts to sustain the interest of the students.

The use of arguments reflecting students' common misconceptions, or 'visual disagreements' (Dabell, 2008) to create cognitive conflict (Naylor & Keogh, 2013), is an important strategy for the design of a comics package. As a result, the student will be driven to resolve their cognitive conflict. During the classroom lesson, this provides an opportunity to promote conversations among students to spark new ideas (Dabell, 2008). This is also possibly a crucial step in getting higher achieving students to probe deeper into the concepts (Naylor & Keogh, 2013). In addition, this strategy helps engage students who might not be comfortable speaking up publicly. When students agree with one of the viewpoints in the comics, the threat of being wrong is minimized, which could boost their confidence to speak up (Keogh & Naylor, 2000).

Therefore, in the text component of the framework, the designer should be cautioned that the amount of text in the comics be kept to the minimum, preferably in the form of dialogues with conflicting viewpoints. The crux is to get the concept across while keeping students interested in the storyline and therefore, the storyline should not scare the students away. The conflicting viewpoints will help generate discussions, and the teacher takes on the role of a facilitator, rather than a lecturer.

Humour. Humour could be a very powerful tool in education. According to Grecu (2008, as cited in Shmakov & Hannula, 2010), humor can serve multiple functions, such as motivating students and developing students' critical thinking when used in teaching. Compared to lengthy texts found in traditional textbooks, students are likely to be more interested in reading comics rather than textbook. Humour will help capture students' attention better and increase their uptake of the mathematics content.

In a study by Toh et al. (2019), humour in the comics led the students to engage in more discussion and collaboration with their classmates during lessons. Other studies have shown that humor can also improve students' retention of concepts (e.g., Musa et al., 2020; Menezes et al., 2019; Garner, 2006) and their attention to the instructional material (e.g., Lujan & DiCarlo, 2016). Humour can also help reduce students' defensive attitude towards learning the subject (Menezes et al., 2019) and reduce their mathematics anxiety (Şengül & Dereli, 2010). Mathematics anxiety has also been found to contribute to severe consequences, such as committing elementary arithmetic errors and has been correlated to lower mathematics achievement (Ashcraft, 2002). Therefore, comics are especially useful for low-attaining or unmotivated mathematics students. Therefore, for students who lack interest in Mathematics and find it difficult, humor can be a game-changing factor that helps students enjoy and be engaged in learning Mathematics.

Humour should be used sensitively and should not result in mocking students' involuntary mistakes (Grecu, 2008, as cited in Shmakov & Hannula, 2010). When adding the element of humor, teachers need to be very sensitive to the students. It is important to select materials that match the age and culture of students. As Toh et al. (2016) suggest, mathematics concepts and ideas should be introduced to students in an informal manner. This help to lower students' resistance to reading and learning the concepts. In conclusion, incorporating humor allows comics to be very effective in helping students move from simpler to more difficult concepts.

Design Booster

In addition to the PATH guidelines, we further suggest additional "design boosters", which we labeled the CoHANa (Colours, History, Assessments, Naming) guidelines, which have been shown to provide benefits to students' learning. Together, the two guidelines form the PATH-CoHANa framework.

Colors in the comics. Colours have been shown to attract attention, hence improving memory performance (e.g., Farley & Grant, 1976; Pan, 2010; Smilek et al., 2002; Spence et al., 2006). This has been shown in empirical studies to translate to better retention of concepts (e.g., Wu & Dyer, 1986, as cited in Pett & Wilson, 1996). Mack and Rock (1998) found that when people are not paying attention to a visible stimulus, they may not be aware of its presence at all. This can explain why some students do not capture concepts even when they are emphasized in the students' worksheets. Therefore, attracting students' attention to the important concepts in comics through the use of colors is critical. For example, Azamain et al. (2020) used different colors to represent the different speeds in a distance-time graph. The different colour attract students' attention and help them identify the different conditions in speed (stationary, non-accelerating, accelerating). The benefits of using colors in comics have also been echoed by Shahrill et al. (2022) who found that students are more interested and ready to learn in lessons with comics when the comics were colored.

The use of colours, however, should not be excessive. Shahrill et al. (2022) also cautioned that the colo rs and characters in the comics may overly attract students' attention and distract them instead. Regarding the use of colors, Faiola and DeBloois (1988) suggested that the number of colors used should be kept to about six, which is what the eye can keep track of at a single glance. They suggest

avoiding complementary colors (e.g., red, and green) on the same screen. Also, they suggest using commonplace color codings, such as red for danger and warnings. Lastly, the text and background color should have high contrast, such as black text on white, to ensure readability.

History of mathematics. The use of the history of mathematics to teach mathematics has been promoted by educators for many decades (Lim & Chapman, 2011). Several studies have shown the benefits of using the history of mathematics in the teaching of mathematics. For example, Swetz (1986) suggested that educators can turn to the history of mathematics as a source of mathematics problems. He argued that these problems not only help to improve students' mathematical problem-solving skills but also illustrate how the problem-solving process has evolved. This is corroborated by Fried (2001) who suggested that mathematics history provides a context for mathematics problems and alternative strategies for problem-solving.

According to Lim and Pang (2002), their students appreciated the integration of the history of mathematics into mathematics lessons as it gives meaning to the mathematics that they are learning in class. This is especially important for weaker students who might perceive mathematics as meaningless algorithms. Lim and Chapman (2015) carried out lessons with the use of the history of mathematics in their study and found positive effects on students' mathematics achievement. Students also liked the lessons as the history aspect introduced to their applications of mathematics, made the lessons fun and enjoyable, and help them better understand the origins of mathematical formulae and notations. Therefore, incorporating the history of mathematics into teaching mathematics has potential benefits that will help students appreciate mathematics more.

However, some students might dislike the lessons that incorporated the history of mathematics as well (e.g., Lim & Chapman, 2015). The major complaints included the irrelevance of mathematics history to their examinations, and the long time needed to go through the historical contents. Some students found the lessons difficult to understand and boring. These are problems that can be ameliorated with the use of comics. By condensing the long historical contents into short stories that are presented in comic form, students will spend less time reading a long chunk of texts. Furthermore, presenting the concepts found in these historical contents in the graphical form will help students understand them more easily. Therefore, comics are a great strategy that allows the integration of the history of mathematics into classroom instruction.

There are already attempts by authors at incorporating the history of mathematics into current textbooks. In a textbook written by Yeap et al. (2021), they provided short stories, notes, and questions on the origins of concepts. For example, on the topic of volumes and surface areas, a short story and question based on how Archimedes determined if the king's crown was made of pure gold, was added to the back of the chapter. However, these stories are often seen as extra materials that are often omitted in classrooms. Therefore, for students to gain the benefits of learning from mathematics history, it must be able to capture the interests and attention of students. Therefore, such historical stories can be presented to students in the form of comics which will make reading them greater pleasure than reading big chunks of text.

Assessment. Apart from conveying mathematical concepts in a simplified manner to students, comics be effective assessment tools, for both formative (Chin & Teou, 2009) and summative assessments (Martinez, 2004). It was found that when assessing students with comics and cartoons, students found the assessment less threatening, and students were more willing to explain their choice of answer compared to the traditional modes of assessment (Martinez, 2004). Since mathematics anxiety can often lower students' competency in mathematics (Ashcraft, 2002), integrating comics into mathematics assessments can bring great benefits for students.

In typical classrooms, students take on a passive role in learning, where most parts of the lesson involve students listening to the teacher and answering questions posed by the teacher. Students do not get much opportunity to understand mathematical concepts when they are passively listening (Takahashi, 2009). Comics, on the other hand, have the effect of making students take on an active role. When they agree with the belief of a character, they have a motivation in defending their opinions. During the discussion, they have to challenge alternative ideas as well (Chin & Teou, 2009). As a result, comics can trigger a lot of cognitive conflicts and thinking as their perceptions are challenged by others. Hence, teachers gain access to the misconceptions that students might generally not be willing to disclose.

Assigning names to characters. Assigning names to characters in the comics can improve fluency when students conduct discussions on the story and the embedded concepts in the comics. This is especially true in the case of comics with multiple characters. Students will be able to express their agreement with a particular named character, who can be interpreted as embodying a particular concept or viewpoint. In a study by Kabapinar (2009), it was shown that students biased towards or against a particular comic character did not affect their reasoning. Furthermore, assigning names to the characters could make the comics more authentic and has a personal touch for the students.

An exemplar of a comics-based instructional material

In this section, we exemplify the use of the PATH-CoHANa framework in a sample comics-based worksheet for the lesson on the volume of cones and spheres. Both sets of comics were colored to improve their attractiveness to the students.

The first set of comics portrayed Benjamin and Sharmaine who went on a shopping trip. They were getting ready for a party and had to buy some conical party hats. The scenario was modeled closely after plausible students' encounters in daily life, including the jokes that Benjamin made. Benjamin's antics of wearing a party hat too small for his head and Sharmaine picturing him as an ice cream cone are also attempts at infusing humor into the comics. The texts used to convey the story and the associated mathematical concepts in the comics are kept to a minimum. Lastly, we incorporated practice questions for students to work out in the comics, allowing them to apply the new formula they just learned from the comics. This is followed by a few practice questions with an increasing level of difficulty.

In the second set of comics, we used the famous story of how Archimedes found the method of displacement to determine if the king's crown was made of real gold. This was followed by his discovery of the sphere's volume. The story serves to engage students and spark their interest in mathematics through the use of casual history of mathematics. With the text in the comics kept to a minimum, the students would be able to get the gist of the story quickly. The comics were also followed by practice questions for students to work on.

The inclusion of the two comics helps to bring life to these two mathematical concepts. We believe that comics can certainly be a powerful force in helping students enjoy learning mathematics.

Conclusion

In this paper, existing literature based on using comics for education was reviewed. Using the findings from these papers, we synthesized the PATH-CoHANa framework, consisting of a set of principles that educators and designers can follow when designing comics-based instructional material on mathematics. The guidelines are general and can be applied to different ages. A set of comic strips was designed based on the framework for the lesson on volume.

More educators have started to appreciate the power of comics in teaching and learning. Its charm can potentially attract the attention of students and address the cognitive and affective needs of students in learning. This is related to their motivation in learning. We acknowledge the limitation of the report as the resource has not been trialed in authentic classrooms. Nevertheless, readers who are mathematics teachers are encouraged to adopt or adapt the resource for their classroom instruction.

Acknowledgement

The authors would like to thank Ms. Alison Ang for illustrating the comics strip that is presented in this paper.

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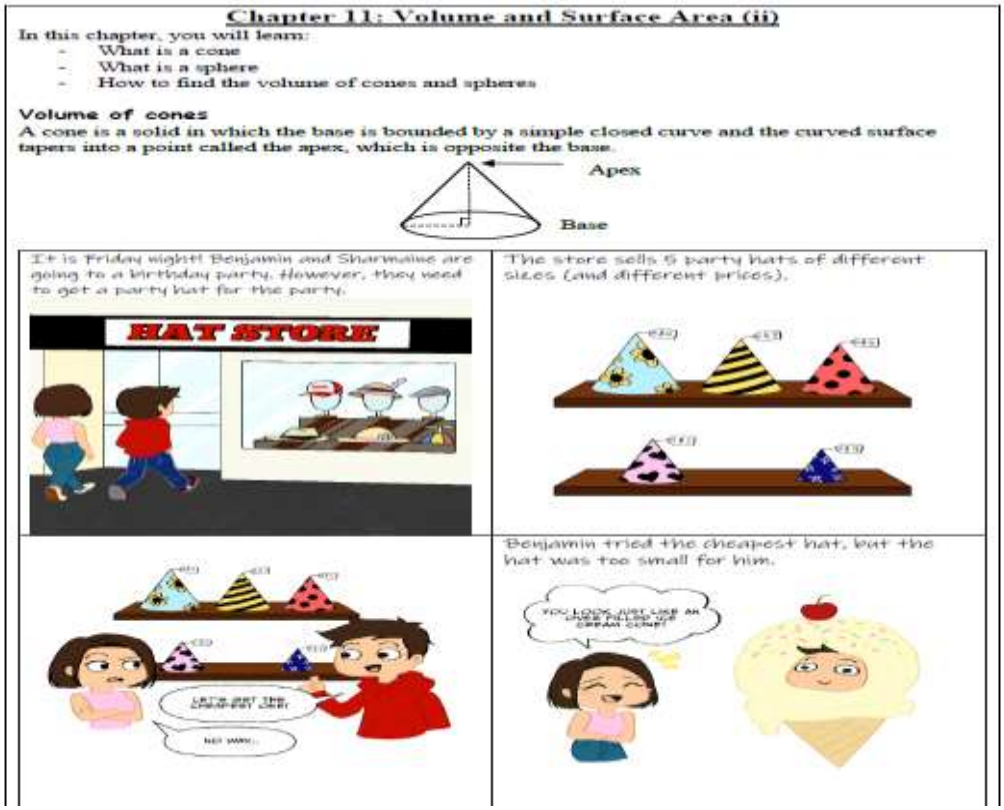
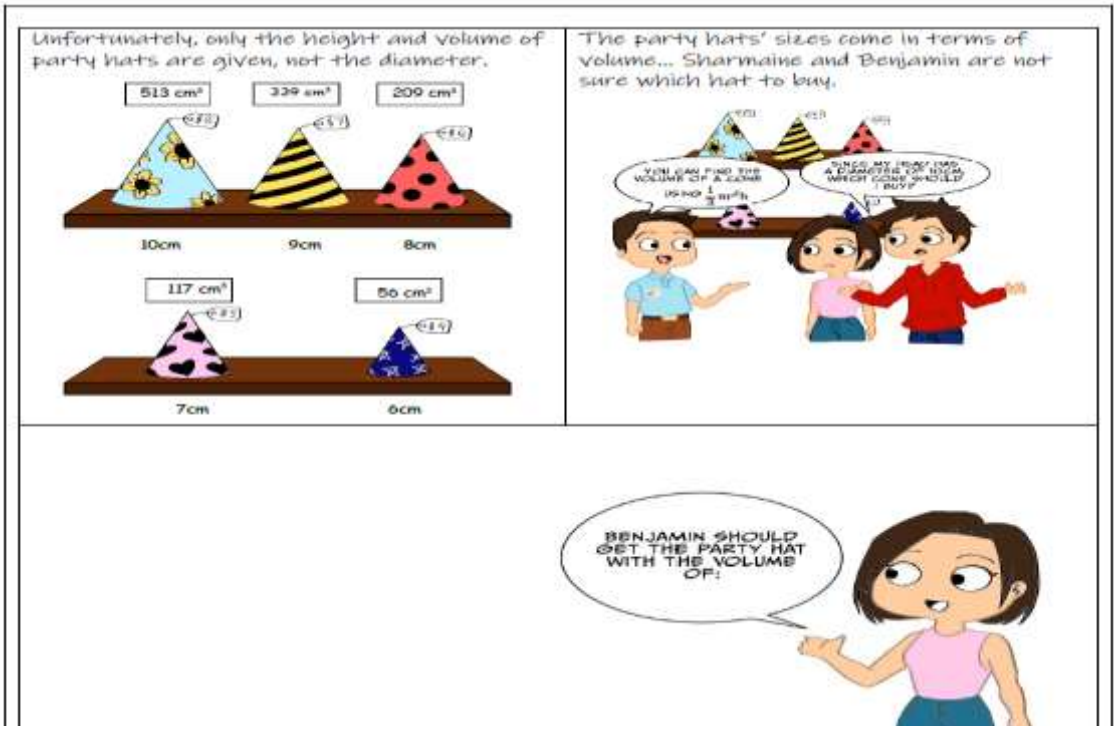
Appendix A

An exemplar of a comics-based instructional material designed according to the PATH-CoHANa framework





Volume of cones - Practice

Question 1
A cone has a height of 17cm and a volume of 445 cm^3. Find the radius of the circular base.

Question 2
A cone has a circular base of radius 4cm and a height of 25cm. Find the volume of the cone.

Question 3
A conical block of copper has a height of 8cm and a base radius of 20cm. A blacksmith melts the copper to craft coins that are 0.3cm thick and 1.2cm in diameter. Find the number of coins that can be crafted.

Question 4
A bifrustum (Figure 1) can be made by connecting the base of two frustum. Each frustum is made by cutting off the top of a circular cone as shown in Figure 2. Find the volume of the bifrustum.

Figure 1

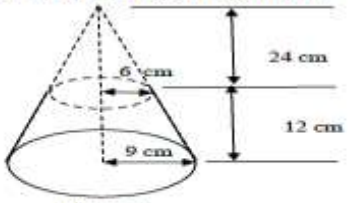


Figure 2

Volume of spheres
A sphere is the 3-dimensional geometric version of a circle. It is also the shape of a ball.

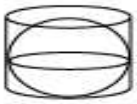
There are many stories about the discovery of the volume of a sphere. Below is one of the many versions of the discovery.

In the 200 BC lived Archimedes, a great mathematician. The king summoned Archimedes.

While taking a bath, Archimedes thought of an idea all of a sudden.

He placed the crown in a beaker of water and obtained the volume of the water displaced. As a result, he was able to find the volume of the crown.

Supposedly, the volume of the crown/ water displaced was 550 cm^3, and the mass of the crown was 10kg. Since the density of gold is 19.3 g/cm^3, was the crown made of pure gold?

Following the discovery of the displacement method, Archimedes applied this principle to discover the formula to find the volume of a sphere. 	He found that the volume of water displaced by the sphere was $\frac{2}{3}$ the volume of the cylinder. Can you work out the formula to find the volume of a sphere? 

Volume of spheres - Practice

Question 1

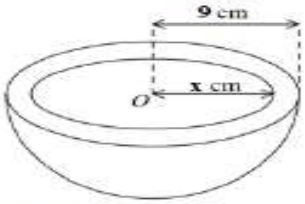
A ball has a radius of 0.3cm. Find

(i) the volume of the ball.

(ii) The mass of 10 balls if they are made of iron, which has a density of 7.9 g cm^{-3}.

Question 2

A hemispherical bowl with centre O has an outer radius of 9 cm and an inner radius of x cm.



(a) Calculate the volume of the external hemisphere.

(b) Find the value of x if the volume of the bowl is 117 cm^3.