DEVELOPING FRACTION SENSE USING VIRTUAL MANIPULATIVES

Ronny Kwan Eu Leong
Universiti Malaya
<rkleong@um.edu.my>

Abstract

Learning fractions has been one of the challenges for many students in elementary mathematics. This study was carried out to see how students' informal knowledge and prior knowledge influence their conceptual meaning of fraction when they used virtual fraction manipulative for learning. Virtual manipulative tools were used as they are regarded as essential cognitive technological tools. Clinical interview was employed as a form of assessment of thinking. It was used in the beginning of the lesson to gauge the student's existing ideas on fractions. The instructional method involved interactions between the teacher and the students while solving fractions tasks. Finally, a closing interview was conducted to evaluate whether the lesson promoted the student's understanding on fractions. The researcher achieved most of the goals for this lesson. In addition, the lesson using virtual manipulative was fairly effective in promoting the conceptual knowledge of fractions.

Introduction

Fraction is used when a quantity is smaller than the unit or as a ratio to compare two quantities. According to Filep (2001), the concept of natural numbers originated from counting while the concept of fractions was derived from measurement. Studies (Behr, 1983; Mack, 1995; Moss, 2005) have shown that students have difficulties with fractions. The difficulty in learning to use fractions is much greater than the difficulty of learning to use natural numbers (Nunez & Bryant, 2008). One of the causes of this problem is that fraction consists of many sub-constructs such as the part-to-whole comparison, a decimal, a ratio, an operator, an indicated division (quotient) and a measure of continuous or discontinuous quantities (Behr, et al., 1983). As such according to Kieran (1976), to completely understand fractions, one must know the sub-constructs and their interrelations (Kieran, 1976).

Understanding Fractions

The meaning of fractions to many children is typically influenced by their informal knowledge. Most of the time, the symbolic representation of fraction is closely related to the student's prior knowledge of whole numbers (Mack, 1995). It is important for children to learn to connect quantities that must be represented by rational numbers with their mathematical symbol and notation. In this study, the researcher focussed on students' understanding of fractions with limited prior knowledge. Furthermore, this study investigated how students use their informal knowledge to build an understanding of the fraction as a part of the whole.

Virtual Manipulative

Technology has played an important role in making mathematics lesson interesting and more enjoyable. With technology, teachers are able to develop their mathematical instructions. One of the new categories of manipulatives that is available in the World Wide Web is the virtual manipulative. A virtual manipulative is "an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard, & Spikell, 2002, p.373). It is also known as "computer based renditions of common mathematics manipulatives and tools" (Dorward, 2002, p. 329). Some concrete manipulatives such as Cuisenare Rods and Tangrams are available at no cost on the World Wide Web. One of the benefits of the virtual manipulatives is that it allows the user to manipulate them and this is a very useful feature in having an interactive mathematics classroom.

Virtual manipulatives are also regarded as cognitive technological tools (Zbiek, Heid, Blume, & Dick, 2007). The ability to go beyond the capabilities of the physical manipulatives is one of the features that make the virtual manipulatives unique. Many classroom research actions have identified that the use of the virtual manipulatives is more complex due to the unlimited objects (Behr, 1992 & Moyer, 2002). However, this helps develop conjecturing and exploration of mathematical concepts as virtual objects can be changed to different shapes, sizes and colours that a concrete object is unable to do. With the learners' ability to explore the different kinds of representations and making connections between them, meaningful learning of Mathematics can be achieved. Students usually struggle to make a connection between the visual and symbolic representations and visual manipulatives can help students overcome it. A student may be able to develop more complex understandings of concepts when using virtual manipulatives (Moyer, Niexgda & Stanley, 2005).

Literature Review

A classroom study of third graders that was done by Moyer (2005) indicated that virtual manipulatives helped over half of the students in the class to improve their conceptual understanding of fractions and also their conceptual knowledge. There were four consistent benefits of the manipulatives: the virtual manipulative helped the students' to learn fraction, the virtual manipulatives were easier and simpler than paper-and-pencil tests, the advantage of receiving immediate feedback from virtual manipulative applets, and the enjoyment of using the manipulative.

Clinical interview method was used as a form of assessment to enhance thinking. It is essential to know what the child thinks of fractions. Obtaining the right answer in fractions should not be the main objective of a lesson but instead the reasoning of a child that should be valued. The understanding of fractions by a student is explored in this study. Clinical interview was used in the beginning to know how much the student knew about fractions. After the lesson, the students were asked questions on fractions as a whole and what they understood from it.

The instructional method used was direct teaching but in this study it also involved a lot of interactions between the teacher and the students. The research aims to investigate the relationships between the parts and whole of the fraction. This will be done by using the virtual fraction model. This research cannot capture the understanding of a students' mind. To explore students' knowledge on fractions, questioning technique was used while interviewing students. According to the New York State Standard in Grade 3 Mathematics,

manipulatives, visual models, and illustrations should be used to name and represent unit fractions, as part of a whole or a set of objects (New York State Learning Standards).

Research Aims

This study was done to see how informal knowledge and prior knowledge of students influenced their conceptual meaning of fraction with the usage of virtual fraction manipulatives. In addition, we wanted to find out the extent to which students understood the concept of fraction as part of a whole. Previous studies involving virtual manipulatives indicated good results on student achievement (Moyer, Suh, & Heo, 2004), increase in confidence, the simple user friendly usage and in addition enjoyment for users (Reimer & Moyer,2005). The theoretical framework of this study was based on by Behr's (1983) rational number concepts and Mack's (1995) whole number and fraction concepts when building on informal knowledge.

Methodology

In this study, the researcher used the virtual manipulatives of the **National Library of** Virtual Manipulatives for Interactive Mathematics (NLVM) and the Twin Cities Public Television (TPT) to teach the concept of fraction. TPT collaborated with the Rational Number Project (RNP) and University of Minnesota to develop the Teaching Fractions Project into a media enriched online with funds from the US Department of Education's Ready to Teach programme in 2002. What convinced the researcher to use the fraction applet from TPT was the modelling of the concrete material; instead of using the hands-on materials like paper, virtual version of papers were used. By doing the paper folding virtually, this offered the students unlimited exploration opportunities. The visualization aspect of the fraction with the virtual paper is very impressive. In addition, the researcher also used a task from the Illuminations website from National Council of Teachers of Mathematics (NCTM) as a teaching tool as well as to gauge students' understanding of fractions. Studies had shown that it is very useful for students to use virtual pattern blocks in solving problems and reasoning with fractions (Reimer, 2005 & Zbiek, 2007). By using virtual fractions, students are able to investigate the relationship between parts and wholes using a different kind of representation of a region model.

Subject

The interview subject was a third grader Hispanic student, Justin (pseudonym). He loved Mathematics because it was fun and challenging. However, at times he disliked it when he did not understand a concept. Justin is of average mathematical ability and has some basic knowledge of unit fractions and formal instruction on fractions.

Testing conditions

The researcher was grateful that Justin's parent consented to the interview with Justin. The interview was conducted at the library of Columbia University in New York and it lasted for about two hours. Justin was more than willing to answer all my questions on fractions. He also seemed very excited in the beginning and also mentioned that he was ready for the interview. Towards the end, even though Justin seemed tired but he tried his best to answer all my questions and he was co-operative. The researcher realised that Justin was comfortable

talking and expressing his ideas on fractions and this created a good atmosphere for the session.

Lesson Plan

This lesson has four objectives. At the end of the lesson, students will be able to:

a) identify fractions when the whole (region) and a part of the region are given
b) represent the fractional relationship between the pattern block shapes
c) identify and understand the numerator in a fraction
d) identify and understand the denominator in a fraction

In addition, the researcher also investigated the effectiveness of using virtual manipulatives in teaching children on the conceptual knowledge of fractions. The lesson was divided into 3 sections. In the first section, the researcher tested Justin's knowledge about fractions. This was done by using questions and figures as shown in the Result section. Generally the open ended question probed on:

a) representation of fraction
b) symbolic meaning of fraction
c) understanding of fraction
d) examples of fraction
e) application of fraction

During the second section, the researcher taught the subject about fraction as parts of a region using the virtual manipulative applet from TPT. Then he used the virtual pattern block from the NVLM to further illustrate examples on fractions as a part of a whole. While showing the examples, the researcher asked Justin to answer some questions to test his conceptual knowledge of fractions. At the end of the lesson, the researcher gave Justin a worksheet on fractions. Justin explained aloud why he had written down each answer and he was to explain some of his answers.

In the final section, the researcher did a clinical interview to find out what Justin knew about fractions after the lesson with the virtual fraction. The questions focussed on:

i. representation of fraction
ii. interpret whether the diagram given is a fraction
iii. the meaning of the numerator and denominator of a fraction
iv. understanding on fractions as a part of a whole

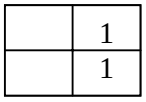
Results and interpretation

At the start of the interview, the researcher told Justin to answer the questions to the best of his abilities. It was not so much focused on getting the right answers but more on identifying Justin's thinking process. The questions asked were simple but Justin must explain the answers aloud. Justin seemed to understand the instructions, so the researcher started the lesson.

Firstly, the researcher asked Justin, "How do you divide a pizza into four?" Justin then drew a pizza and divided it into four parts. The next question which was "Can you draw for me one half?" A triangle was drawn by Justin and he added a vertical line in the middle to separate it into two parts. I asked him "What does this part represent?" while pointing to one half of the triangle. He said "One". Next I showed him the symbol of ½ and asked "What is the name of

this symbol?" He mentioned that it is one half. I moved on with "Why is it called one half?" His answer fascinated the researcher: Justin said "The number on top is supposed to be one, say you have a ruler and this is in the middle of one and two". The researcher asked Justin to illustrate what he said aloud with a diagram. Justin pointed to the middle of the number one and two on the scale and mentioned half was located there (after drawing the scale on a ruler). The researcher felt that Justin got the idea of one half but when he represented it on a scale, he indicated that the half was on 1, that is, half way between 0 and 2. Perhaps the term ½ that consist of the number one and two contributed to his understanding on the representation of half on a number line. Below were the questions the researcher asked Justin on fractions before the lesson began.

Question : Do you know the name of ½?
Answer : Fractions.
Question : Shows ¼. What is it called?
Answer : One forth
Question : Shows 1/8. What is it called?
Answer : One eighth
Question : Explain to me what you understand with ¼?
Answer : ¼ is a number. Number can be in a lot of stuff.
Question : What does one here represent? (Numerator of ¼)
Answer : One in fraction represents one half, the number that is split up into one or two.
Question : Show me the diagram of ¼.
Answer : Draws a square with four partitions and labels it with ones.
Question : Where is ¼?
Answer : Point to one part of the square.
Question : How many parts are in this diagram?
Answer : Four
Question : What about the number one on top?
Answer : It could be 2 numbers, one on two, or two on one.
Question : What can you say about the 3 fractions ½, ¼ and 1/8?
Answer : One of these is in the middle of 2 numbers.
Question : What can you tell me about a fraction?
Answer : Fraction can be used for measuring.
Question : Some examples?
Answer : You can measure a pencil and if it is little, it is in the middle of 2 numbers right there on the line and is one half.



From the line of questioning above, it seemed clear to the researcher that Justin had a limited conceptual understanding of fractions. There seemed to be two possibilities to his understanding on fractions: a) he understood that half could be represented on a number line , in this case using the example of the ruler b) he was able to show the partitioning of unit fractions like one half and one forth with different representations . His answers were quite consistent on what fractions are even though the researcher asked him the same questions in a different way. Justin also used his informal knowledge of shapes, like a triangle and rectangle in this case to represent a fraction. It was encouraging that he was able to use other shapes besides the common circle to represent a fraction. When asked what one fourth was, Justin did say that it was a number. The researcher should have probed further with the question "Why is it a number?" Perhaps he was confused with the term. As Justin was explaining the numerator of one forth, he mentioned about two numbers and he related it with one or two.

The researcher assumed that Justin was referring to the fraction one half because that was the basic fraction that he knew very well. When asked other questions about fraction, Justin replied that it was used for measuring. He then related an example of how it could be used.

Next the researcher started his lesson on fractions. He used the virtual fraction applet from TPT to show Justin some examples of fractions. The fraction applets model the concrete object of a folded paper. This was very useful because the researcher could show Justin many different kinds of fractions that were very colourful. While teaching, the researcher applied the questioning technique. Justin was asked to answer several questions. The researcher helped explain the correct solutions if Justin did not get it right. Part of the dialogue during the lesson is shown below:

Question : I showed him a rectangle with two parts.
How many parts were there?
Answer : Two
Question : If I were to shade one of the parts, what is the fraction?
Answer : One half
Question : What does one half mean?
Answer : It's like the parts in the diagram.
Question : What does one (referring to the numerator of one half) mean?
Answer : One part
Question : Coloured or non-coloured?
Answer : The one represents the non-colour.
Comment : No, one represents the coloured part. Then I explained that one is the shaded part and two is the total equal part.
Question : A new rectangle is divided into three parts. How many parts are there?
Answer : 3.
Question : If I were to shade one part of the fraction, what is the fraction?
Answer : 1/3
Question : Two parts?
Answer : 2/3
Comment : Next, I showed Justin a rectangle divided into four equal parts. I then explained to Justin that the whole is a rectangle and it is broken into four equal parts. Next I shaded one part of the diagram and asked him, "What is the fraction?"
Answer : One forth.
Question : Why one forth?
Answer : One part is shaded so it is one forth.
Question : What is two forth mean?
Answer : Two refers to the two shaded parts and the bottom is how many parts are there
Question : Why is the fraction of the diagram 2/8?
Answer : The number on the bottom equals to eight parts. The number on the top is the coloured sections.

Next, the researcher used the virtual pattern block by the National Library of Virtual Manipulatives (NLVM) to guide Justin through a fraction activity (refer to http://illuminations.nctm.org/LessonDetail.aspx?ID=L344). Justin was shown how to manipulate the virtual fraction and how to use it. Then, Justin was asked to read aloud the question "How many green triangles are in one blue rhombus?" The researcher then dragged

two green triangles one at a time to fill the blue rhombus. Justin was asked to answer and he said "Two green triangles." Next, he researcher asked Justin to write down his answer in the worksheet that had been provided. In the next question, Justin was asked, "The green triangle is what fraction of the blue rhombus?" He mentioned two. In this case, the question was not clear to Justin. The researcher explained to Justin that the question wanted to know what was the fraction of one green triangle from the whole diagram? He understood my question and said the answer was one half. The pattern block helped Justin to see a fraction as a part of the whole and he thought it was fascinating. The different colours and shapes gave him an idea that fractions had many different representations. Justin seemed to benefit from the usage of the pattern block to understand fraction.

After the lesson the researcher gave Justin a worksheet on fractions. The task was for him to name the fractions from the given shaded rectangles. After writing down each answer, Justin was asked to explain how he obtained his answers. For example in the first question, the diagram shown was 3/5, and Justin wrote it down correctly as 3/5. I asked him "Why is it 3/5?" He answered that "Because you are shading 3 and the total is 5". Then in the next question that showed a fraction of 7/8, Justin wrote down 7/1 as his answer. The researcher asked Justin how he obtained his answer. Justin replied that "7 is coloured and 1 is left, so seven and a half, but no, it should be seven ones." I then asked him "Why one?" Justin gave the same explanation that only one part is not coloured. The researcher probed Justin further "What is the number of the total parts? What is the number of the shaded parts?" Justin mentioned eight for the total parts and seven as the shaded parts. At this time, he was quite restless and couldn't concentrate on the questions. When asked what was the fraction again, he mentioned seven one. Justin was asked again "Earlier, what did you tell me about the total parts?" Justin mentioned "Eight". Then the researcher said "What is the fraction now?" Justin responded with "one eighth". I asked "Are you sure? What is the fraction again?" and Justin responded with his answer "seven eighth". The researcher felt that Justin must have realised his mistake as the researcher was asking him the same questions many times. He must have felt that his answer was incorrect as the researcher kept asking him questions instead of informing him that his answer was wrong and he must have sensed his mistake.

In the next session, the researcher continued with some questioning using the clinical interview method. I told Justin "Draw for me a diagram to represent ¾"? He first drew a circle and partitioned it into four parts. He then shaded one part of it. Justin was asked to explain his answer. He then realised his mistake and instead shaded three parts. The researcher probed Justin by asking what did the three in ¾ mean? Justin replied, "The shaded parts". Next question, Justin was asked about the meaning of four. Justin said that it referred to the total parts. Then Justin said, "Oh, I realised my mistake" and he continued shading two more parts of the circle so that the total shaded parts was three. Perhaps Justin was so used with the unit fractions where he would usually only shade one part for the numerator. However, he realised his mistake after being asked the meaning of the four parts. He might also have felt the mental fatigue because this was after 50 minutes of lesson.

Next Justin was tested on his understanding of fraction. The researcher drew a circle and shaded one portion. Then the researcher asked Justin "Can I get a fraction from this diagram?". Justin replied that he could not do it because it was not right. Then he proceeded by drawing another line on my diagram. Next, the researcher asked Justin to show him another diagram where a fraction could exist. Justin then drew a rectangle and divided it into four equal parts and shaded three parts of it. Justin realised that "Why can't I get a fraction from my diagram?" His answer was that "There is too much split. There can be only four

splits in a certain shape". The researcher further said "What about the parts?" Justin replied "It doesn't exist. Only in certain shape you can do it." It is obvious that Justin knew that a fraction could not exist by just partitioning the shape. He was unable to offer an explanation why it was not a fraction other than relating it to the shape of the object. Perhaps, he was more concerned about the representation than the equal part of each partition must have to form a fraction. With more practise, he should be able to have a more in-depth knowledge on fractions.

Reflection

Overall, I was satisfied with my teaching. But there are few areas I can improve further. In the preparation section, I have read many articles to help me select suitable virtual fractions for my lesson. I also have written down all the questions I could think of for the clinical interview. Even though it was a one-to-one teaching, it was quite challenging but enjoyable. During the teaching, I realised in some instances I spoke a little too fast while giving the instructions and I must give a longer waiting time for my subject to respond after asking a question. Sometimes I gave hints to the question too quickly and repeated my questions a few times. There were a few instances where after Justin gave an answer I did not ask him "How did you get that" or "How did you know that?" Some of these things affected my instructional goal. In terms of the usage of the virtual fraction, I demonstrated how to use it and also allowed Justin to explore his understanding of fractions by using it. At the end of the day, I have done my best and I hope to do better in my next task.

Conclusion

In short, the researcher was glad that he had achieved most of the goals for his lesson. From the lesson and clinical interview, Justin was able to

a) identify fractions when the whole (region) and a part of the region were given
b) represent the fractional relationship between the pattern block shapes using a standard form of the written notation of fraction
c) identify the numerator in a fraction and understand that the numerator is the top number in a fraction and indicated the number of parts of the whole even though he made some mistakes in some of the tasks
d) identify the denominator in a fraction and understand that the denominator is the bottom number in a fraction and indicated the number of parts into which the whole is divided even though Justin got it mixed up with the numerator

In addition, the researcher found this lesson to be fairly effective in teaching the basic conceptual knowledge of fractions by using virtual manipulatives. Furthermore, the researcher observed that Justin really enjoyed exploring the fraction tasks by exploring the virtual fractions in NVLM.

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