BASIC EDUCATION COMMON CORE REGIONAL LEARNING STANDARDS (CCRLS): STEP TOWARDS PARALLELIZING SEAMEO MATHEMATICS CURRICULAR FRAMEWORK WITH EXEMPLARS

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Abstract

Many countries in South East Asia came out among the bottom third in TIMSS 2011 and PISA 2012 in both mathematics and sciences. There is no agreement on the scope, breadth, width and content of basic education among SEAMEO member countries. Every country has different learning standards in mathematics subject. The purpose of this paper is to elaborate on the stages of drafting Southeast Asia Basic Education Standards (SEABES) Common Core Regional Learning Standards (CCRLS) in mathematics so that the countries can use this as a guide in order to benchmark with the curricula of high performing countries in the international student's assessments as well as analyzing other international curriculum documents from countries like Singapore, South Korea and Japan. This will help to create the parallelization of the curriculum provisions across countries and learning expectations as well as learning outcomes that will enable students to contribute productively to their individual country and the region as a whole. The method used is content analysis. A mapping of the curriculum was done and many series of workshops were organized in RECSAM, Penang Malaysia. Two regional seminars were organized in RECSAM and University of Tsukuba during the last 2 years. The findings and the result will be highlighted and reported on how these documents could be used and accepted/endorsed by all SEAMEO member countries as a reference for mathematics curriculum development and revision.

Keywords: Curriculum; Mathematics; Basic education; Mathematics curriculum framework; Mathematics learning outcomes; Sustainable development.

Introduction

Mathematics is in the unenviable position of being simultaneously one of the most important school subjects for today's generation. Its reputation is awe-inspiring. Everybody knows how vital it is and everybody knows that they have to study it. But few people feel comfortable with it; so much so that it is socially quite acceptable in many countries to confess ignorance about it, to brag about one's incompetence at doing it, and even to claim that one is math phobic! Do we really know what we should be doing? These basic questions become even more important when consideration is made in the context of the growing problem areas. The first is the concern felt in many countries about the direction which mathematics education should take in the face of the increasing presence of computers and calculator-related technology in society (Bishop, 2013).

Regional integration is the process by which two or more nation-states agree to co-operate and work closely together to achieve peace, stability and wealth. It is also a process in which

neighboring states enter into an agreement in order to upgrade cooperation through common institutions and rules (Carleton University, 2016).

The statement above is suggestive to the 'ASEAN community' for establishing the opportunity for the development of an educational policy framework for all SEAMEO Member countries in order to enhance access to educational opportunities, to support the development of quality basic education and to encourage regional mobility. Such a framework will support all Governments as the main providers of basic education to meet the learning needs of all students. Further, the SEAMEO Education Agenda #7 "Adopting a 21st Century Curriculum" states the need to pursue a radical reform through systematic analysis of knowledge, skills, and values needed to effectively respond to changing global contexts, particularly to the ever-increasing complexity of the Southeast Asian economic, socio-cultural, and political environment, developing teacher imbued with ASEAN ideals in building ASEAN community within 20 years (2015-2035) (MOE, 2013)

Research Questions

The purposes of this paper are to elaborate the development of stages of SEABES in mathematics so as to strengthen ASEAN collaboration across different educational system to effectively respond the changing global context and complexity around ASEAN countries and to provide world-class learning standards in Mathematics, including 21st century skills that can be used as benchmarks in SEAMEO Member Countries to ensure all students have access to fundamental knowledge, skills and values in order to be socially responsible, globally competitive and sustainable.

Through this way, SEAMEO defines basic education standards as the competency in teaching and learning of science and mathematics that are achieved through engagement in education in order to function successfully in society taking into account cultural differences and the importance of ASEAN values.

The following are research questions identified:

(1) What are the common Content Strands for Mathematics curriculum in Key Stage (KS) 1 to 3?

(2) How could these be applied to plan Mathematics lessons with exemplars showing how spiral curriculum or learning progression can be implemented?

(3) What are the uncommon standards of curriculum in SEAMEO but well implemented after curriculum mapping?

Trends and Issues in Mathematics Education

In this information society, The role of mathematics increases including to establish 21st century skills through reviewing mathematics as the patterns of science for future prediction and design of big data which stimulates innovation not only for technology advancement but also for business model. Mathematics is an essential subject to establish common reasoning for sustainable development of society through viable argument in understanding each other and developing critical reasoning as the habit of mathematics.

Many countries in South East Asia came out among the bottom third in TIMSS 2011 and PISA 2012 in both mathematics and sciences. Even though the students like mathematics and

sciences, their international performance is always susceptible because of many factors. According to PISA and TIMSS report by MOE Malaysia, we lagged behind in 3 major things: Our teachers failed to execute their full potential in giving high order thinking skills level question because they used to focus on questions that tailor to exams and nation building (unity and harmony). Our students did not use to reading long questions where they have to extract important facts and then solve the questions. Our text books and curriculum are quite new and the questions are not fully tested yet (Masami, 2012).

Building on the strengths of any curriculum which will prepare the next generation and that will bring tremendous opportunity, especially in Asia, SEABES will also bring many changes that we cannot foresee today. The task of our schools and tertiary institutions is to give our younger generation the chance to develop the skills, character and values that will enable them to continue to do well and to take forward in their future.

Lately, Singaporean curriculum has been moving towards an education system that is more flexible and diverse. Flexibility and diversity of the curriculum use provide students with greater choice to meet their different interests and ways of learning. Being able to choose what and how they learn will encourage them to take greater ownership of their learning. These were also giving students a more broad-based education to ensure their all-round or holistic development, in and out of the classroom (Ministry of Education Singapore, 2016).

This approach in education allows nurturing the young with the different skills that they need for the future, seeking to help every child find his own talents, grow and emerge from school confident of his abilities. This will encourage them to follow their passions, and promote a diversity of talents among them – in academic fields.

Curriculum planners must act fast to remediate declining quality in education specially on the curriculum planning and implementation because if we are not careful, our students might face the consequences where they fail to enter top universities in the world and may be high end companies will shut their doors for our Malaysian graduates and semi-skilled workers (Serif, Ibrahim, Abdullah, & Yusof, 2016).

Methodology

The method used is content analysis and participative inquiry. A mapping of the curriculum was done and many series of workshops were organized in RECSAM, Penang Malaysia. Two regional seminars were organized in RECSAM and University of Tsukuba during years between 2014 and 2016.

The CCRLS were developed based from the strengths of the existing national education standards of the SEAMEO Member Countries. The various activities undertaken in the development of the draft of working paper of CCRLS in Science and Mathematics include:

- curriculum review and comparison of the national education curriculum of the seven SEAMEO Member Countries in Science and Mathematics, namely Brunei Darussalam, Cambodia, Indonesia, Malaysia, Philippines, Singapore and Thailand;
- identification of similarities and differences in terms of content/domain/topics/strand by country;
- tracking of content/domain/topics/strands across grade levels from the primary to secondary level; and

- consolidation of content standards and performance standards by subject and by grade levels from primary to secondary levels.

The consolidated SEAMEO Mathematics Standards were benchmarked with the standards of high-performing countries in international student assessments such as Hong Kong, Japan, Australia, United Kingdom and US; as well as relative documents such as 2015 TIMSS Framework and National Council of Teachers of Mathematics (NCTM) as well as research studies and literature that are available on what students need to know and be able to do to be successful in college, career, and life.

Figure 1 shows the flow process of the development of the Common Core Regional Learning Standards (CCRLS) in science and mathematics (Mangao, Suhaidah, & Mohd Johan, 2005, p. 3).

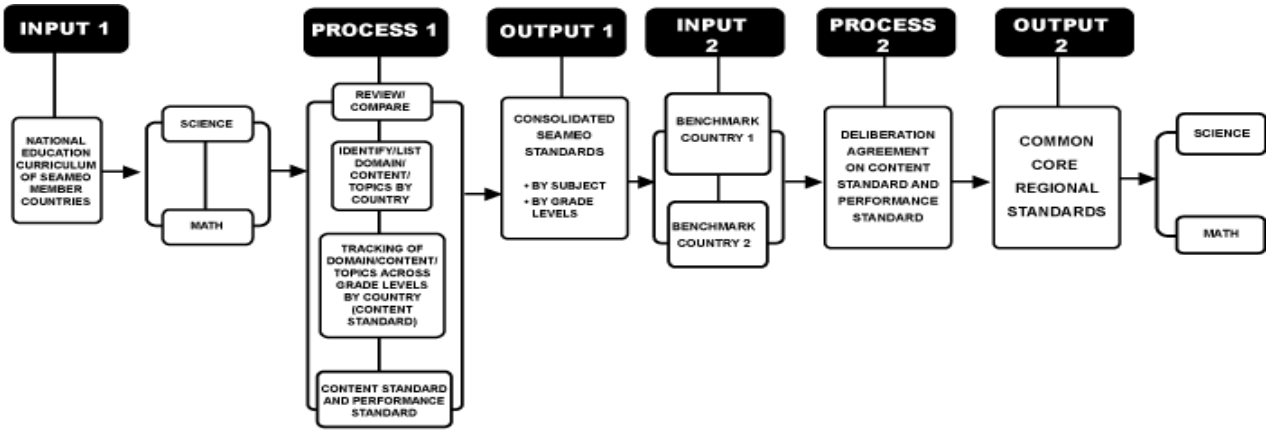

Figure 1. Framework in developing the CCRLS in Science and Mathematics for SEA-BES (Mangao et al., 2015, p. 3).

Series of SEA-BES Workshops

A series of workshops which aims to develop the draft of the Common Core Regional Learning Standards (CCRLS) in Mathematics were conducted on the different dates, levels and venues as follows:

A. National Level (RECSAM in Penang, Malaysia)
   - 2 April 2015 (In-house/Local Level, SEAMEO RECSAM, Malaysia)
   - 11 May 2015 (In-house/Local Level, SEAMEO RECSAM, Malaysia)
   - 21-22 May 2015 (In-house/Local Level, SEAMEO RECSAM, Malaysia)
   - 23 July 2015 (In-house/Local Level, SEAMEO RECSAM, Malaysia)
   - 27 August 2015 (In-house/Local Level, SEAMEO RECSAM, Malaysia)
   - 17 September 2015 (In-house/Local Level, SEAMEO RECSAM, Malaysia)
   - 27-28 January 2016 (In-house/Local Level, SEAMEO RECSAM, Malaysia

- 24-25 March 2016 (In-house/Local Level, SEAMEO RECSAM, Malaysia

B. Regional Level
- 4-5 November 2014 (Regional Level, SEAMEO RECSAM, Malaysia)
- 20-22 October 2015 (Regional Level, SEAMEO RECSAM, Malaysia)
- 15-18 February 2016 (Regional Level, University of Tsukuba, Japan – Only for Mathematics Standards)
- 13-18 June 2016 (Regional Level, University of Tsukuba, Japan)

Participation and Involvement of Experts and Educators

Maximum participation and involvement of experts as well as teachers across the Southeast Asian region and beyond were solicited in the development of the draft CCRLS. Their tasks include giving inputs and providing specific, constructive feedback on the draft standards being a member of curriculum working group. The following groups were involved: (Mangao et al., 2005, p. 4):

- consultants (Professor Masami Isoda, Center of Research on International Cooperation in Educational Development, University of Tsukuba, Japan; Dr. Mark Windale, Centre for Science Education, Sheffield Hallam University, United Kingdom; and Professor Kerry J. Kennedy, Hong Kong Institute of Education, Hong Kong)
- curriculum experts in Mathematics from the 11 Ministries of Education of SEAMEO Member Countries,
- SEAMEO Secretariat
- SEAMEO centers (i.e. RECSAM, QITEP in Mathematics, SEAMOLEC)
- local educational institutions (i.e. USM, IPG)
- Science and Mathematics national centers (i.e. IPST, UPNISMED)
- Mathematics Master Teachers from Penang State (Master teachers and IPG lecturers)

Data Analysis, Results and Discussions

Curriculum standards in every country are written under specific format for all school subjects under the regulation and policy of the ministry in every country as well as with direction of curriculum reform. The format itself embedded the direction of national reform of movement itself.

SEABES CCRLS Mathematics are developed considering the 21st century skills with format chosen to distinguish and relate the descriptions of standards beyond the stages. With regards

to this format, every standard embeds the ideas of what are expected to be learned, how these are related with the standards of each other and why it is necessary for further learning.

Common Core Regional Learning Standards (CCRLS) is not the curriculum itself but is a key reference for further collaboration of the curriculum development, assessments, and professional development on the demands of 21st century skills which are related with SEAMEO Priorites No.5 and 7. National Curriculum Standards in all SEAMEO member countires which are established in every country are respected. In response to this demands, CCRLS is prescribed for curriculum developers and teachers educators.

The Common Core Regional Learning Standards (CCRLS) in Mathematics encompasses three key learning stages of schooling, namely; Key Stage 1 – lower primary level (Grade 1 to Grade 3); Key Stage 2 - upper primary level (Grade 4 to Grade 6); and Key Stage 3 – lower secondary level (Grade 7 to Grade 9). Following the description of every stage will support the discussion on meeting the basic proficiency level of mathematics knowledge which is necessary in the preparation for continuous learning in their life.

Key Stage 1 serves as the foundation of knowledge covering the basic facts and skills developed through simple hands-on activities, manipulation of concrete objects, pictorial and symbolic representations. This stage focuses on arousing interest, enjoyment and curiosity in the subject through exploration of pattern, characterization, identification and description of shapes, performance of four fundamental operations, identification of its algorithm, understanding of basic mathematical concept and skills experienced in daily life. Calculation of quantities will also be established to carefully and willfully understand the attribution of objects that are used to make direct and indirect comparison.

Key Stage 2 builds from the competencies acquired from the first stage. This stage provides the extension of numbers, measurement and relations, plane figures and solid figures as well as data representations and graphs. This level will enable deepening of experiences in their life and allow the use of mathematical terminologies, performing investigations as well as establish the base for analyzing, evaluating and creating. Appreciating the beauty of structure of mathematics will enable them to enjoy and sustain the learning.

Key Stage 3 builds from the competencies acquired from the second stage. This stage discusses number and algebra, relations and function, space and geometry as well as statistics and probability. This presents higher cognitive demands dealing with abstract ideas and concepts that enhance critical thinking, creative thinking through the application of knowledge and understanding of abstract concepts as well as principles in daily life in participating discussions, dialogue, arguments pertaining to contemporary societal, economic, technological, political, environmental and mathematical issues. The knowledge enables the creation of better future predictions for the betterment of the living environment. This knowledge also bridges the further mathematics learning in different job demands.

The Content Strands

The arrangement of major learning elements in Mathematics is now put into four content strands in every key stage, in total of twelve content strands. In every stage, four content strands are mutually related. The same content strand names are not used to indicate progression beyond each stage. For example, in Key Stage 1, ‘Numbers and Operations’, Key Stage 2, ‘Extension of Numbers and Operation’ and Key Stage 3, ‘Numbers and Algebra’. The name of the content strands is progressing beyond the stages showing the extension and integration of

content. In the case of measurement, Key Stage 1 relates with quantity and setting of units. In Key Stage 2, it extends to non-additive quantity beyond dimension. In Key Stage 3, measurement is not used as one content strand because the idea of unit is embedded everywhere. For example, square root is an irrational number which means unmeasurable; Pythagorean Theorem in geometry used for measuring, proportional function is used for counting the number of nails by weight, and in statistics, new measurements units are expressed such as quartile for boxplot.

Strands are used to explain mutual relation of content (Kilpatrick, Swafford, & Findell, 2001). The term 'Domain' is sometimes used for compartmentalization though categorization of content.

Appendix A and B summarized some of the possible application of CCRLS in the planning of spiral curriculum for mathematics lesson. The following are some discussions made.

Key Stage 1
- Numbers & Operations
- Quantity & Measurement
- Shapes, Figures &Solids
- Pattern & Data Representations (Refer Appendix A)
- Mathematical Process & Humanity

Key Stage 2
- Extension of Number & Operations
- Measurement & Relations
- Plane Figures & Space Solids
- Data Representations & Graphs (Refer Appendix A)
- Mathematical Process & Humanity

Key Stage 3
- Numbers & Algebra
- Space & Geometry
- Relationship & Functions
- Statistics & Probability (Refer Appendix A)
- Mathematical Process & Humanity

Appendix A illustrates an example of how spiral curriculum or learning in progression can be prepared for in-depth study on mathematics focus on Key Stage 1 Strand 4 'Pattern and Data Representation', Key Stage 2 Strand 8 'Data Representation and Graphs' as well as Key Stage 3 Strand 12 'Statistics and Probability'.

Appendix B is an example of how CCRLS can be used as reference to prepare mathematics lesson with suggested activities that promote mathematical processes, higher order thinking and humanity.

Analysis of Key Stages for the Application on Lesson Planning
The following analysis of Key Stages serve as guide for the planning of mathematics lessons.

Numbers and Operations for Key Stage 1. Number is introduced with situations, concrete objects, pictorial and symbolic representations. The ways of counting and distributions are extended through addition, subtraction, multiplication and division. The Base Ten Numeral System is the key roles for extending the numbers and operations for standards algorithm in vertical form. Procedure of calculations and algorithms are also focused. Through the establishment of fluency of calculations with connection of situations, number sense will be developed. Fraction and decimals are introduced with manipulative.

Quantity and Measurement for Key Stage 1. Attribute of objects are used to make direct and in-direct comparison. For comparison, the non-standard and standard units are used. Counting activities denominate units of quantities such as cups for volumes, arm-length and hand-spans for length. Standards units such as m, cm, kg and L are introduced. Time and durations which are not based ten systems are introduced. Money is used in everyday life which is not complete model for based 10 systems. Through the calculation of the quantities, the concept of conservation of the quantities will be established. The sense for quantity is developed through the appropriate selection of measurement tools.

Shapes, Figures and Solids for Key Stage 1. The basic skills of exploring, identifying, characterizing and describing shapes, figures and solid based on their features are proposed. Activities such as paper folding enable exploration of various features of shapes. Identification of similarities and differences about shapes and solids enable classification to be done for defining figures. Using appropriate materials and tools, relationship in drawing, building and comparing the 2D shapes and 3D objects are considered. Through these activities, the skills for using the knowledge of figures and solids will be developed. The compass is introduced to draw circles and take the same length.

Pattern and Data Representations for Key Stage 1. Various types of patterns are treated such as the number sequences and repetition of shapes. Size of pictures can be represented by the number sequences. Tessellation of shapes and paper folding can be represented by the repetition of shapes. Exploration of pattern and features are also considered to represent data structure in our life with pictograph. Patterns and features produce meaning and represents mathematical information. Patterns are represented by diagrams and mathematical sentences which are also used for communication in identifying and classifying situations to produce meaningful interpretations.

Extension of Numbers and Operations for Key Stage 2. Numbers are extended to more digits, fractions and decimals. Multiplication and division algorithms are completed and acquired fluency. Fraction becomes numbers through the redefinition as a quotient instead of part - whole relationship. Multiplication and division of decimals as well as fractions are also explored to produce the ways for calculation. Various representations are used to elaborate and produce meaning for the calculation. Number sense such as approximate numbers, relative size of numbers and value are enhanced for the practical reasoning in the appropriate context of life.

Measurement and Relations for Key Stage 2. Additive quantity such as angles, areas as well as volume and relational quantity such as population density, and speed are introduced. Additive quantity can be introduced by establishment of the standard unit which is the same way as the quantity and measurement of Key Stage 1. Relations of quantities in situations are discussed with the patterns such as sum is constant, difference is constant, product is constant

and quotient is constant using tables as well as represented by mathematical sentences and letters. Proportion and ratio are introduced with representations of diagrams, graphs and tables for multiplication, and connected with decimal and fraction. Percents are introduced with diagrams in relation to ratio and proportion. Relational quantity is produced by different quantities with understanding of ratio. Areas of a circle are discussed through a proportional relationship between radius and the circumference. Idea of ratio and proportion are fluently applied for real world problem solving.

Plane Figures and Space Solids for Key Stage 2. For example, through tessellation, figures can be extended through plane figures. Parallelogram and perpendicular lines are tools to explain properties of triangles and the quadrilaterals as plane figures. For topics e.g. identifying and recognizing symmetry and congruence also need parallelogram and perpendicular lines. Plane figures are used to produce solid in space and vice versa. Opening faces of solids would produce plane figures which are referred as nets. Activities related to building solids from plane figures are emphasized and encouraged because finding the area of a circle through numerous sectors of the circle to construct a rectangle. Circles are used for explaining the nets of cylinders.

Data Representations and Graphs Key Stage 2. The process of data handling is introduced through data representation such as table, bar graph, line graph, bar chart and pie chart. The graphs are utilized depending on the qualitative and quantitative data used such as bar graph to distinguish category and count in every category. The discussion of producing the line graph includes taking data at specific intervals, suitable scale used and slope. Histogram which is necessary for reading the data representation of social study and science is also used as special type of bar chart. Averages are introduced based on the idea of ratio for making the variability of histogram evenly, and used for summarizing and comparing data on table. Logical analyses to understand the whole possible cases such as tree diagram are introduced for knowing the ways to represent logical reasoning.

Numbers and Algebra for Key Stage 3. Numbers are extended to positive and negative numbers, and square roots. Algebraic expressions are already introduced by the mathematical sentence and symbols at Key Stage 2. At Key Stage 3 algebra are operated by expressions and equation until the second degree. On the extension of the calculation from numbers to symbolic algebra, reasonable and meaningful ways of calculations are established in various possibilities. Like and unlike terms are introduced in an algebraic sentence and expression, and introduce property of equations, then operate equation fluently and expansion and factorization enabling further operations of the polynomial. Substitution, addition and subtraction of equations enabling further operation for equations.

Relationship and Functions for Key Stage 3. Relationships are represented by equations and system of equations. Functional relations are treated amongst situation, table, equation of function are introduced based on patterns as well as relations with algebraic representation on Key Stage 2 and Key Stage 3. Solution of simple equation is done by equivalence deduction based on algebra learnt earlier. Two variables in simultaneous equations as simple system of equations are solved by substitution and additive-subtractive methods. Three representations, table, equation and graph, are used as methods to analyze the properties of every function. Direct and indirect proportions are redefined with those representations mentioned. Direct proportional functions are extended to line functions. The comparison of indirect proportion and line functions makes clear the property of linearity with ‘constant ratio of change’. The concept of proportion is extended to function of y = a (x^2). Ways of translations between table

and equation, equation and graph, graph and table are specific skills for every function with fluency.

Space and Geometry for Key Stage 3. Space and Geometry provides the ways of reasoning for exploring properties in geometry and produce the ways of argument to explain justifications of visible reasoning. The calculations of angles are not just simple calculation but also the ways of using the geometric propositions to justify answers through explaining why it is correct based on basic properties. Explaining the relationship of figures using transformation, identify the properties of congruency and describing similarities. Calculation of angles and proofs are means for developing the habit of reasoning on plane figures. The conditions of congruence and similarities, properties of circles, are also used to explain and prove appropriateness of geometric conjectures in relation to triangle, squares, and circles. Dynamic geometric software as well as simple compass and ruler are used for exploration of assumptions. It is also well synchronized with searching general idea by special cases such as counter example.

Statistics and Probability for Key Stage 3. Data handling are extended to exploring the viability of histogram with mean, median, mode and range. Exploratory analysis of given data using ICT is enhanced. Histogram shows the different viability if we change the class. Probability is introduced as ratio with the law of large number. Sample space with assumption of equi-probability becomes the point of discussion. Histogram can be seen relatively and produce frequency distribution polygon. Difference between sample and senses is also discussed. Boxplots with quartile that is an extension of median and range is also used for the comparisons of distributions.

Integration of Mathematics Processes and Humanity Strand: Processes, Values, Attitude and Ways of Thinking

In mathematical process, there are three components which are composed of Mathematical ideas, Mathematical Thinking and Mathematical Activities. Humanity encompasses three components such as Mathematical Values, Mathematical Attitudes and Habits of minds for citizens. Mathematical process and humanity strands bridge the standards in different strands for learning of process skills and human values. The context will be simplified by the following three processes: developing mathematics, argumentation for understanding others and sophistication through critique, and applying mathematics through modeling and replacement. Extension and generalization is a key for developing mathematics. Producing understandable explanations are usually related with understandable representation such as diagram and materials for showing simple structure. Example is used for demonstration by the specific case and counter example is used for checking generality. Process of modeling usually includes problem formation, solving mathematically, and explanation of the meaning. On those three contexts, values of mathematics such as recognizing beauty of pattern are learned through the appreciation of mathematical experience. Appreciation of others' idea which includes understanding ideas with sense and meaning.

Mathematical Processes and Community for Key Stage 1, Enjoyable mathematical activities are designed to bridge the standards in different strands. Exploration of various number sequence, skip counting, addition and subtraction operations help to develop number sense that is essential to support explanation of contextual scenarios and mathematical ideas. Mathematical ways of posing questions on the life are also necessary learned at this stage. Ability to select simple, general and reasonable ideas enables effective future learning. Application for number sense provides facility for preparing sustainable life. The use of ICT tools and other technological tools provides convenience in their life. At initial stage, concrete

model manipulation is enjoyable, however drawing diagram is the most necessary for explaining complicated situations by simple way of representation.

Mathematical Processes and Community for Key Stage 2, As a follow up of Key Stage 1, activities are designed to enable an appreciation of knowledge learned and the ways of learning such as application of knowledge of number sense to solve daily problems. Mathematical processes such as communication, reasoning are utilized to provide explanation for mathematical problems and modelling. The ability to connect and reason mathematical ideas would trigger an excitement among learners. Discussions of misconceptions are usually enjoyable and challenging. Mathematics learning usually begins from the situation at Key Stage 1. On Key Stage 2, the development of mathematics is possible through the discussion for the extension of the forms. Appreciation of ideas and representations learned become part of the enjoyable activities. Through the consistent use of representation such as diagram, applications of learning become meaningful.

Mathematical Processes and Humanity for Key Stage 3, Enhance the logical argument in mathematics for communication with others. Proposed challenging activities are used to promote thinking at different level of arguments to make sense of mathematics. Translating real life activities into mathematical model and solving problems using appropriate strategies are emphasized in functional situations. The processes of doing mathematical activities involve patience that develop perseverance in learners and take responsibility of one's own learning. At this stage, the habitual practice of self-learning will eventually develop confidence, thus, opportunity for challenges to extend mathematics and the ability to plan sequence of future learning are also enhanced.

Findings on In-depth Analysis of Key Stages and Well Established SEABES CCRLS
Specifically, the following were the findings of documentary analysis under Key Stage 1 which are not common standards for South East Asian countries but well established after workshops conducted to develop SEABES CCRLS:

The denomination such as 3 cups, 2 cups, one cup is described at strand on the quantity and measurement. Unit for 'counting' is used for describing in the quantity and measurement. Number pattern is discussed under pattern and data representations. One million is too big for 'counting' and is introduced only for learning of the three digit system. Understanding the relationship of 'addition and subtraction' is discussed under pattern and data representations. Money system is discussed under 'measurement'. Meaning of area is use to describe at the measurement and relation. Denomination is necessary for learning the group of counting. It is also describe in pattern and data representations and number and operation. Calendar is possible in the Key Stage 1 under pattern and data representation. Coin and currency is depending on the country. Some countries use currency by twenty. Pattern of shapes is discussed under 'pattern data and representation'. Number sequence will be under number and operation. Time and duration are discussed under Quantity and Measurement. Harmony of shapes will be under 'shapes, figure and solids'. Denomination will be under quantity and measurement. It is related with number and operations, quantity and measurements as well as pattern and data representations. STEM education is enhanced. Mathematics is the major and base subject for STEM Education. Hence, technological contents are included in mathematics.

The following were the findings under in-depth analysis of Key Stage 2 which are not common standards for South East Asian countries but well established after workshops conducted to develop SEABES CCRLS:

Billion is too large for counting and is introduced in the three digit system under relative size of number. Metric system and names of units are discussed under measurement and relations. Under the three digit system, if we teach until thousandths we can extend by three digits. In measurement and relations, utilization of constant sum, difference, product and quotient are described (e.g. 25 -21 = 4, 26 – 22 = 4, 27 – 23 = 4). Discussions of decimals until hundredths are related to the use of money. It is a minimum requirement. If we teach until hundredths and further place value students can understand.

Applying the idea of multiplication into ratio, percent and proportion is discussed in measurement and relations. Extension of fraction more than one is done by using the fraction with quantity for situation such as 4/3m. Selecting the appropriate denomination of quantities and units for fraction in the context, e.g. 2/3m is two of 1/3m and the whole is 1m, however 3/3m is 3x(1/3)m; the structure is the same as tens is ten of units and discussed under 'measurement and relations'.

Applying the idea of the multiplication of fraction into ratio, proportion, percentage and base is discussed in measurement and relations. Right angles are already done at Key Stage 1 on Shapes, Figures and Solids for explaining the properties of Figures. Conservation of angles is to be re-learnt at triangle under key stage 2 under plane figures and space solids. Constants of multiplication and division, corresponds proportionality in multiplication table at Key Stage 1 under number and operations. Constant of addition and subtraction are treated in Key Stage 1 under pattern, and data representation. A Place folder is introduced in Key Stage 1. Multiplications were studied in Key Stage 1 number and operations. On number and operations Key Stage 2, rate is the value of division as quotient. Band graph and pie chart are discussed under data representations and graph. Ratio of different quantity is rate. Ratio of the same quantity is narrow meaning of ratio. The value of fraction as ratio is not necessary a part of whole in situations. Fraction as ratio is usually used in the context of multiplication in situation where denominator is the base or unit for comparison. Enlargement is under plane figures and space solids. The graph is under data representations and graph. Proportion and Inverse proportion is necessary in Key Stage 3 in science. Relationship on polygons and circles are under plane figures and solids. Ratio and rate is discussed under measurement and relations. Proportions are learnt under measurement and relations. Ratio is learnt under measurement and relation. How to draw histogram is under statistics and probability. Social studies and science subject on reading histogram is necessary. Using ICT for drawing graph will be mentioned at mathematical activities strand. Mean is introduced as average under measurement and relations. Employing the 'Problem- Plan –Data Analysis Conclusion' (PPDAC) itself will be described at the mathematical activities later. Ratio and proportions bridge multiplication and division in situation of two quantities with reference to extension of number and operations and measurement and relations

The following were the findings under Key Stage 3 which are not common standards for South East Asian countries but well established after workshops conducted to develop SEABES CCRLS:

Simple case of polynomials may vary from and depending on the countries based on the mapping of curriculum. Pythagoras Theorem is discussed in Space and Geometry strand. The graph of quadratic equation will be treated in the function and relation. Utilizing ICT is recommended in mathematical activities. Family of linear functions is recommended to use ICT tool under mathematical activities strand. Utilization of the concept and ideas on

relationships and functions are used as ways of mathematical modeling in their life. Simple geometric construction is discussed by the ruler and compass with reasoning. Dynamic geometric software usually draws entire circles for identifying in variant activities. It is useful to find the invariant properties which are discussed in mathematical process and humanity. Pythagoras theorem is used for re-understanding the topic on square number and algebra. There are three meanings of form:

1. Permanence of form means "keep the pattern of operation" such as (-3)x(+2)=-6, (-3)x(+1)=-3, (-3)x0=0, and (-3)x(-1)=+3, and (-3)x(-2)=+6. Here, the product of the pattern increases by 3.
2. The form means "Principle of the permanence of equivalence of form" which means to keep the law of commutativity, associativity and distributivity.
3. The form means the axiom of field in Algebra, normally, in education, we only treat No.1 and No.2 as discussed above. For the extension of the number to positive and negative numbers, beyond the limitations of meaning such as subtract smaller number from larger number. For the extension of numbers to irrational number, beyond the limitation of meaning such as rational number is quotient (value of division).

Conclusion and Recommendations

The SEABES Common Core Regional Learning Standards in Mathematics are still produced as draft document and works that are in progress. Both standards are constantly developing and evolving as current issues and trends are being accommodated. Though substantial work has been accomplished, yet much more need to be done.

In order for this draft working paper of the SEABES CCRLS to be considered world class quality and truly reflecting ASEAN values, more opportunities for in-depth discussion and seeking of agreement from all SEAMEO member countries have to be provided to show ownership and unity of purpose. It is hoped that the SEABES CCRLS in Mathematics could truly be a significant instrument to achieve goals of ASEAN Community in the near future.

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Appendix A

An Exemplar of How the Key Stages and Strands of SEABES-CCRLS are Established as Reference for Preparing Spiral Curriculum in Mathematics Classroom with Topic of Progression

Key Stage 1 Strand 4: Pattern and Data Representations	Key Stage 2 Strand 8: Data Representations and Graphs	Key Stage 3 Strand 12: Statistics and Probability
Topic 4.1: Using patterns under the number sequence[1] Standard 4.1.1: Arranging object for beautiful pattern under the number sequence Learning Standards 4.1.1. i. Know the beautifulness of pattern in case arranging the object depending on number sequence ii. Arrange object according to number sequence to find simple pattern iii. Arrange expressions such as addition and subtraction to find simple pattern iv. Express the representation of pattern using placeholder (empty box) v. Enjoy the arrangement depending on number sequence in their life	Topic 8.1: Arranging tables for data representations. Standard 8.1.1: Arranging the multi categories data into tables for showing data structure Learning Standards 8.1.1. i. Explore how to collect multi category data based on a situation ii. Explore how to arrange and read multi category data on appropriate tables. iii. Appreciate in using multi category tables in situation. Topic 8.2: Drawing and reading graphs for analyzing data. Standard 8.2.1: Drawing and reading line graph for knowing the visualized pattern as tendency.	Topic 12.1: Exploring distribution with the understanding of variability Standard 12.1.1: Exploring distribution with histograms, central tendency to represent variability Learning Standards 12.1.1. i. Investigate histogram to show distribution appropriately using class and range ii. Identify alternative ways to show distribution such as dot plots, box-plot and frequency distribution polygon iii. Investigate central tendencies such as mean, median, mode and their relationship on the distribution iv. Appreciate the analysis of variability through the finding of the hidden structure of distribution on situation

vi. Find the pattern on the number of table such as calendar[1] Topic 4.2: Producing harmony of shapes using patterns[2] Standard 4.2.1: Arranging tiles of different or similar shapes in creating harmony Learning Standards 4.2.1. _: i. Know the beautifulness of pattern in case arranging the object depending on shapes, colors and sizes ii. Arrange object according to shapes, colors and sizes to show pattern iii. Arrange boxes according to shapes, color and sizes to create structure iv. Arrange circles and spheres for designing	Learning Standards 8.2.1.: i. Introduce the line graph based on appropriate situation such as rainfall, temperature,… ii. Distinguish line graph and bar graph and its usage such as increase, decrease, flat, no-change iii. Introduce the graph of proportion using the idea of line graph and read the gradient by constant ratio[4] iv. Appreciate the line graph in situation Standard 8.2.2: Drawing and reading band graph and pie chart for representing ratio in a whole.[5] Learning Standards 8.2.2. _ i. Explore how to scale the bar or circle for representing ratio or percent	using the measure of central tendency, class and range Topic12.2: Exploring probability with law of large numbers and sample space Standard 12.2.1: Exploring probability with descriptive statistics, law of large numbers and sample space. Learning Standards 12.2.1. _ i Experiment with the help of coins and dice to explore the distribution of ratio and understand the law of large numbers ii Use the idea of equally likely for inference to get the value of probability iii Analyze the situation with tables to represent the sample space and using it for predicting with probability iv Utilize various representations such as table, tree diagram , polygon diagram

1 Time and duration are discussed in key stage 1 under Quantity and Measurement
2 Harmony of shapes will be discussed in Key Stage 1 under Shapes figure and solids
4 Proportions are learnt in the key stage 2 under measurement and relations
5 Ratio is learnt under Key stage 2 measurement and relation

v. Enjoy the creation depending on different shapes, color and sizes in their life Topic 4.3: Collecting data and represent the structure Standards 4.3.1: Counting data through categorization for getting information. Learning Standards 4.3.1. _: i. Explore the purpose of why data is being collected. ii. Grouped the data in creating similar attributes on the denomination[3] of categories and count them (check mark and count) iii. Think about what information is obtained from the tables with categories and how to use it.	ii. Use the band graph and pie chart for comparison of different groups iii. Appreciate the band graph and pie chart in situation. Standard 8.2.3: Reading histogram[6] for analyzing frequency distribution. Learning Standards 8.2.3. _ i. Draw a simple histogram[7] from frequency table on situation ii. Read various histograms for analyzing data distribution iii. Use mean[8] to compare different groups in the same situation with histograms Topic 8.3 : Using graphs in PPDAC[9] cycle appropriately Standard 8.3.1: Identifying appropriate graphs for data analysis in PPDAC cycle.	and frequency distribution polygon for finding probability v Appreciate articles for sustainable development with probability to imagine the future Topic 12.3: Exploring statistics with sampling Standard 12.3.1: Exploring sampling with the understanding of randomness Learning Standards 12.3.1. _: i Discuss the hidden hypothesis behind sample and population on ratio ii Use randomness to explain sampling iii Appreciate data sampling in a situation with sustainable development Strand 12: Mathematical Processes and Humanity

3 Denomination will be learned in Key stage 1 under Quantity and Measurement
6 How to draw histogram is discussed in Key stage 3 under statistics and probability. Social studies and science subject on reading histogram is necessary
7 Using ICT for drawing graph will be mentioned at mathematical activities strand.
8 Mean is introduced as average in key stage 2 under measurement and relations
9 PPDAC itself will be described at the mathematical activities later.

Standard 4.3.2: Organizing the data collected and represent it into a pictogram for easy visualization Learning Standards 4.3.2. _ i. Produce the table and pictograms from collected data under each categories ii. Interpretation of tables and pictograms as a simple conclusion about the data being presented. iii. Appreciate pictograms through collecting data and adding data in daily activities in their life Standard 4.3.3: Representing the data structure by using bar graph and using it to imagine the future of communities. Learning Standards 4.3.3. _ i. Understand to draw bar graph from table under data categories and sort the graph for showing its structure ii. Appreciate the ways of presenting data such as table, pictograms and bar	Learning Standards 8.3.1. _ i. Critique the situation and discuss the expectation before taking data for proper clarification of the objective ii. Plan the survey for the expectation iii. Take the data based on the objective from the situation iv. Utilize appropriate graphical representation which is most suitable for the objective v. Appreciate the graph before taking the conclusion Topic 8.4: Applying Data Handling for Sustainable Development Standard 8.4.1: Applying data handling for Sustainable Development[10] and appreciate the power of data handling for imagining the future. Learning Standards 8.4.1. _:	12.1 Enjoying problem solving through various questioning for extension of operations into algebra, space and geometry, relationship and functions, and statistics and probability 12.2 Enjoying measuring space using calculations with various formulas 12.3 Producing proof in geometry and algebra 12.4 Utilizing tables, graphs and expressions in situations 12.5 Using diagrams for exploring possible and various cases logically 12.6 Exploring the graph of functions by rotation, by symmetry and by translation of proportional function 12.7 Understanding the ways for extension of numbers 12.8 Designing sustainable life with mathematics 12.9 Utilizing ICT tools as well as other technological tools

10 This standard which is related to SDG as inter subject content between social studies and science

graphs with sorting for imagining their future communities Strand 4: Mathematical Processes and Humanity 4.1 Enjoying problem solving through various questioning for four operations in situation 4.2 Enjoying measuring through setting and using the units on situations 4.3 Using blocks as models and its diagram for performing operations in base ten 4.4 Enjoying tiling with various shapes and colors 4.5 Explaining ideas using various and appropriate representation 4.6 Selecting simple, general and reasonable ideas which can apply for future learning 4.7 Preparing sustainable life with number sense 4.8 Utilizing ICT tools such as calculators as well as other technological tools such as clock	i. Read the data for sustainable development such as emergency preparedness and resiliency , food and energy security, world weather warming, inclusion and human connectivity in society, and lifelong learning in changing society such as TVET (Technical and Vocational and Training) and adopting positive view for changing the society. ii. Understand the idea of probability as ratio and percentage in reading the data for sustainable development such as weather report and risk analysis iii. Experience a project of reasonable size over data handling purposes for sustainable development and appreciate the power of data handling. Strand 8: Mathematical Processes and Humanity 8.1 Enjoying problem solving through various questioning for extension of operations into decimals and fractions with proportionality and new quantities such as area and volume 8.2 Enjoying measuring through setting and using the area and volume on situations	

	8.3 Utilizing using ratio and rate in situations 8.4 Using number lines, tables, and area diagram for representing operations and relations in situations 8.5 Establishing the idea of proportion to integrate various relations with consistency of representations 8.6 Enjoying tiling with various figures and blocks 8.7 Producing variable explanation based on established knowledge, shareable representations and examples 8.8 Experiencing 'Problem-Plan-Data analysis-Conclusion' (PPDaC) cycle in simple projects in their life 8.9 Preparing sustainable life with number sense and mathematical representations 8.10 Utilizing ICT tools as well as other technological tools	

Appendix B

How Can a Teacher Use CCRLS as Reference to be Incorporated in Lesson Plans?

Mathematics

Key Stage: 2 and 3
Time: at least 50 -80 minutes
Time frame: 2 sessions

Common Core Regional Learning Standards

Strand 8 and 12	**Data Representations and Graphs; Statistics and Probability**
Topic 8.2	Drawing and reading graphs for analysis of data
12.1	Exploring distribution with the understanding of variability
Standard 8.2.3	Reading Histogram to analyse frequency distribution
12.1.1	Exploring distribution with histograms, and central tendency to represent variability
Learning Standard 8.2.3.i	Reading various histograms for analysis of data distribution
12.1.1.iii	Investigating central tendencies such as mean, median, mode and their relationship on the distribution
Mathematical Processes and Humanity 8.7	Producing variable explanations based on established knowledge, shareable representations and examples
8.9	Preparing sustainable life with number sense and mathematical representations
12.8	Designing sustainable life with mathematics
8.10 and 12.9	Utilizing ICT tools as well as other technological tools

Lesson Objectives:

Students will:

- develop an understanding of the concepts of mean, median and mode for group and ungroup data, and how these can be used to describe a set of data in various histograms;
- develop an understanding of how the measure of central tendency can be applied in real world situations;
- build data literacy skills by using visual plots to analyze data, interpret results and draw conclusions;
- explain the findings by visual presentations using software.

Rationale

Students are often asked to calculate values of the measures of central tendencies (mean, median and mode). The calculations can be straightforward decision that measures of central tendencies can be used as a description of the typical value of a set of data. Teacher can either use Problem-based learning or Project-based learning depending on which he will target on achieving the outcome. Further, presentations of outcome can be shared by using any dynamic

software available in various histograms for analysis and interpretations. On that process, students will gain a deeper understanding of the advantages and the disadvantages of using mean, median and mode to express the central tendency of a set of data.

Suggested Activity 1: Content knowledge activity

How do you spend your time?

In 2005, The U.S. Bureau of Labor Statistics conducted a survey of 13,000 households asking how the residents spent their time. The results are the American Time Use Survey (ATUS, 2005).

The following table shows the average hours per week day spent by high school students in various activities:

Average hours per day	Employed	Not employed
Educational	5.7	6.1
Work and related	1.9	0.0
Sleeping	8.3	8.8
Socializing, relaxing, and leisure	2.8	3.7
Sports, exercise, and recreation	0.7	0.9
Religious, spiritual, and volunteer	0.4	0.3

NOTE: Data include persons ages 15 to 19 who were enrolled in high school.
Weekdays include non-holiday weekdays during Jan. - May 2005 and Sept. 2005 - Dec. 2005. Data are annual averages for 2005.

- ✔ Could you identify the mean, median and mode for employed and not employed?
- ✔ Conduct a survey of your peers. Ask for similar data.
- ✔ Create a spreadsheet to generate charts that show the data and present your work.
- ✔ Rank your priorities for your time using relevant software. (e.g. Microsoft Excel, ..)

Suggested Activity 2: Provision for life skills or Higher Order Thinking Skills through Outdoor Study

- ✔ Conduct a survey on how a group of people spend their time. Gather data, analyse, present and share the findings highlighting the mean, median and mode of time spent in the form of various histograms
- ✔ Answer the following questions:

  Why would business owners want to change how they spend their time?
  Why would a teacher change the attitude of how kids spend their time?
  Why would a club advisor, coach or music director try to change the kid's free time behavior?

Suggested activity 3: Provision for Values Education through Clearing Views and Ideas in Mathematics

- ✔ Suppose you want to advise on how a teen should change in spending their time in a specific category.
- ✔ Answer the following questions:

Which one would you try to change?
Why would you want to change it?
How would you go about encouraging that change?
Explain in clear, brief, concise and explicit manner using mathematical concepts and ideas.

Suggested Activity 4: Provisions for Integration of Content Areas in Science and Other Related Fields

- ✔ Read the data for sustainable development such as emergency preparedness and resiliency, food and energy security, world weather warming, inclusion and human connectivity in society, as well as lifelong learning in changing society such as Technical and Vocational and Training (TVET) as well as adopting positive view for changing the society. Let them react expressing the data found in the article.